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INNER SUBGROUPS OF FINITE GROUPS

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§ 1. Introduction

Let G be a finite group, and H be a subgroup of G. We call H an inner subgroup of G, when every automorphism of H can be extended to an inner automorphism of G. The purpose of this paper is to prove the following theorems.

THEOREM 1. *Let G be a finite group. If every subgroup of G is an inner subgroup of G, then G is isomorphic to the symmetric groups S_1, S_2 or S_3.*

THEOREM 2. *Let G be a finite solvable group. If every abelian subgroup of G is an inner subgroup of G, then G is isomorphic to the symmetric groups S_1 S_2, S_3 or the quaternion group Q_8.*

THEOREM 3. *Let G be a finite group, and S be a 2-Sylow subgroup of G. Suppose every abelian subgroup of G is an inner subgroup of G, and every abelian normal subgroup of S is cyclic. Then G is isomorphic to the symmetric groups S_1, S_2, S_3 or the quaternion group Q_8.*

It is interesting to classify all finite groups whose every abelian subgroup is an inner subgroup. It seems that S_1, S_2, S_3 and Q_8 are the only examples of such groups. A more interesting problem is to classify all finite groups whose every cyclic subgroup is an inner subgroup. This condition is equivalent to that the character values of the groups are all rational. But this problem seems to be very difficult.

Theorem 1 is a corollary of Theorem 2 and the following proposition.

PROPOSITION 1. *If every Sylow subgroup of a finite group G is inner subgroup of G, then G is metacyclic.*

This proposition will be proved by using a theorem of Gaschütz on automorphisms of p-groups. In the proof of Theorem 2, we shall use Theorem 3.

In the following, every notation is standard and can be found in [1]. Only elementary results are assumed, in paticular Sylow's theorem, properties of p-groups and automorphisms of abelian groups.

All groups will be finite.

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