M. NAKANO
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SECOND ORDER LINEAR ORDINARY DIFFERENTIAL EQUATIONS WITH TURNING POINTS AND SINGULARITIES II

BY MINORU NAKANO

§ 1. Introduction.

WKB approximations are asymptotic approximations for solutions of linear ordinary differential equations, in some domain of an independent complex variable, as a parameter tends to the prescribed value. The largest of such domains of an independent variable is called a canonical region for the differential equation considered. In the part I [3] we considered the differential equation of the type

$$\varepsilon^2 \frac{d^2y}{dx^2} - p(x)y = 0,$$

where x is a complex variable and ε is a positive small parameter, and we constructed canonical regions for the cases where $p(x)=(x-1)^2/x^3$ and $p(x)=(x-1)^2/x$. In this part we treat two cases one of which is given by $p(x)=(x-1)^2/x^2$, and the other $p(x)=x^\nu-1/x^\mu$ (ν is a positive integer, $\mu=2$). The case $\mu=1$ is treated in [4]. The case $\mu\geqq 3$ is treated elsewhere. In the case where $p(x)$ is of the form $c(x-1)^2/x^2$ or $c(x^\nu-1/x^\mu)$ (c is constant) we can also discuss in the same way. For example, if c is positive, we must only replace ε by $\varepsilon/\sqrt{c}$.

As for the first case we construct canonical regions (§ 2). As for the second case we construct canonical regions (§ 3) and get two different types of asymptotic approximations (§ 4 and § 5) of solutions of the differential equation. Between them there is a relation, and we get the relation, i. e., a matching matrix (§ 6).

§ 2. Canonical regions for the case $p(x)=(x-1)^2/x^2$.

The differential equation corresponding to this case has a second order turning point at $x=1$, a regular singular point at the origin and an irregular singular point at $x=\infty$. In order to decide Stokes curves for the case, let us consider the integral

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