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ON THE EXISTENCE OF A COMPLEX ALMOST CONTACT STRUCTURE

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§ 1. Introduction.

A complex manifold of complex dimension $2m+1$ is said to be a complex contact manifold if it admits an open covering $\{\tilde{O}_i\}$ such that on each $\tilde{O}_i$ there is a holomorphic 1-form γ_i with $\gamma_i\wedge(d\gamma_i)^m\neq 0$ and on $\tilde{O}_i\cap\tilde{O}_j\neq\phi$, $\gamma_i=\tilde{f}_{ij}\gamma_j$ for some non-vanishing holomorphic function $\tilde{f}_{ij}$. In general such a structure is not given by a global 1-form γ; in fact, this is the case for a complex manifold if and only if its first Chern class vanishes [7]. It is also shown in [7] that the structural group of the tangent bundle of a complex contact manifold is reducible to $(Sp(m)\otimes U(1))\times U(1)$. Standard examples of complex contact manifolds are the odd dimensional complex projective space PC^{2m+1}, the complex projective cotangent bundle of a complex manifold, etc.. (See [3], [7].)

On the other hand, some of complex contact manifolds are base spaces of principal fibre bundles with 1-dimensional fibres and real contact 3-structure. A typical example of this is a Hopf map $S^{4m+3}\longrightarrow PC^{2m+1}$. Generalizing this situation Ishihara and Konishi studied in [5] fiberings with 1-dimensional fibres of a manifold with real contact 3-structure and defined in the base space a new structure called a complex almost contact structure. In [2] an equivalent definition is given in terms of global tensor fields. The structural group of the tangent bundle of a complex almost contact manifold is also reducible to $(Sp(m)\otimes U(1))\times U(1)$. The notion of a complex almost contact structure is naturally weaker than that of a complex contact structure. In fact, a manifold with a complex contact structure admits a complex almost contact structure, and the converse is true if the complex almost contact structure is normal [5], [6].

Let M be a complex manifold of complex dimension $2m+1$. Let $\mathcal{O}=\{O_i\}$ be an open covering of M. We say, in this paper, M has a Ω-structure if the structural group of the tangent bundle of M is reducible to $(Sp(m)\otimes U(1))\times U(1)$, that is equivalent to the existence of a local 2-form Ω_i of type (2, 0) on each O_i such that $(\Omega_i)^m\neq 0$ and a non-vanishing function $f_{ij}\in U(1)$ such that $\Omega_i=f_{ij}\Omega_j$ on $O_i\cap O_j\neq\phi$.

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