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PLANAR GEODESIC SUBMANIFOLDS IN COMPLEX SPACE FORMS

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Let M^n and $\bar{M}^{n+p}$ be connected complete Riemannian manifolds of dimension n and $n+p$ respectively. An isometric immersion of M^n into $\bar{M}^{n+p}$ is called a *planar geodesic immersion* when every geodesic in M^n is mapped locally into 2-dimensional totally geodesic submanifold of $\bar{M}^{n+p}$. When the ambient manifold $\bar{M}^{n+p}$ is a space form of constant curvature $\tilde{c}$, K. Sakamoto [7] has showed that such an immersion is an isotropic immersion in the sence of B. O'Neill [6] with parallel second fundamental tensor. Using this fact, he reduced planar geodesic immersions into space forms to full, minimal and planar geodesic immersions of compact rank one symmetric spaces into spheres and obtained

THEOREM A. *Let $f: M^n \longrightarrow S^{n+q}(\tilde{c})$ be a planar geodesic immersion. Then the simply connected Riemannian covering manifold of M^n is a sphere, a complex projective space, a quaternionic projective space or a Cayley projective plane. The immersion is rigid.*

A submanifold M^n of a complex space form $\bar{M}^{n+p}(\tilde{c})$ with constant holomorphic sectional curvature $\tilde{c}$ is called *complex* or *invariant* (resp. *totolly real*) if each tangent space of M^n is mapped into itself (resp. the normal space) by the complex structure of $\bar{M}^{n+p}(\tilde{c})$. A complex submanifold of a Kaehler manifold is also a Kaehler manifold. K. Ogiue [5] has showed that if $M^n(c)$ is a Kaehler submanifold immersed in $\bar{M}^{n+p}(\tilde{c})$ and if the second fundamental form of the immersion is parallel, then either $c=\tilde{c}$ (i. e., $M^n(c)$ is totally geodesic in $\bar{M}^{n+p}(\tilde{c})$) or $c=\tilde{c}/2$, the latter case arising only when $\tilde{c}>0$. Moreover the immersion is rigid. When $\tilde{c}\leqq 0$, E. Calabi [1] proved that if $M^n(c)$ is imbedded in $\bar{M}^{n+p}(\tilde{c})$ as a Kaehler submanifold, then $M^n(c)$ is totally geodesic in $\bar{M}^{n+p}(\tilde{c})$.

In this paper we study a planar geodesic immersion $f: M^n \longrightarrow \bar{M}^{n+p}(\tilde{c})$ of a connected complete Riemannian manifold of real dimension n into a complex space form of real dimension $n+p$ with constant holomorphic sectional curvature $\tilde{c}\neq 0$. When the immersion f is complex or totally real, it is an isotropic immersion with parallel second fundamental tensor. Moreover, if the immersion f is totally real and not totally geodesic, we can reduce the immersion to a full, minimal, planar geodesic immersion of M^n into a real projective space $RP^{n+p}(\tilde{c}/4)$ (resp. a real hyperbolic space $H^{n+p}(\tilde{c}/4)$) when $\bar{M}^{n+p}(\tilde{c})$ is a complex projective

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