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ON INFINITESIMAL AFFINE AND ISOMETRIC TRANSFORMATIONS PRESERVING RESPECTIVE VECTOR FIELDS

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§1. Introduction.

Researches into symmetry of dynamical systems exhibit an increase of interest in automorphisms of geometric objects with some dynamical constraints. Recently, Ikeda and Nishino [1] have studied scalar-preserving isometries of a Riemannian space. In 1968–1974, Katzin and Levine treated projective and conformal transformations related with dynamical structures [2], [3], [4], [5]. The present author [6] has shown that an invariance of the equations of motion in a Riemannian space entails an affine or isometric transformation which preserves a vector field. Interest in the same kinds of transformations comes also from other branch [7]. This paper presents solutions to equations proposed in the previous paper [6] and reveals what Riemannian spaces can admit invariance Lie algebras consisting of the infinitesimal affine and isometric transformations preserving respective vector fields. Global situations will be touched upon.

The settings of the problem are briefly reviewed in Section 2. Section 3 deals with Lie algebras of the infinitesimal affine transformations preserving a vector field. Maximal Lie algebras and the allowed Riemannian spaces will be found out there. Section 4 is concerned with Lie algebras of infinitesimal isometries which preserve a vector field, and with the problem of what Riemannian spaces can admit such Lie algebras. Global structures of these Riemannian spaces will be described. Section 5 is devoted to studying a Lie algebra of dimension lower than that of the considered ones in Section 4; this Lie algebra consists of the infinitesimal isometries preserving a scalar. A case which was out of consideration in [1] will be studied. It will be shown what Riemannian spaces can admit Lie algebras of the infinitesimal isometries preserving a scalar, together with remarks on global situations.

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