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ENTIRE FUNCTIONS WITH THREE LINEARLY DISTRIBUTED VALUES

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1. Introduction. A complex number w will be called a linearly distributed value of the entire function $f(z)$ if there is a straight line L of the complex plane on which all the solutions of $f(z)=w$ lie. For the exponential function, every value is linearly distributed. Conversely, Baker proved that a transcendental entire function for which every value is linearly distributed must have the form $a+b\exp(cz)$, where a, b and c are constants.

In this connection we have shown the following result in our previous paper [4].

Let $f(z)$ be a transcendental entire function. Assume that there are three distinct finite complex numbers a_j and three distinct straight lines L_j of the complex plane on which all the solutions of $f(z)=a_j$ lie $(j=1, 2, 3)$. Assume further that $f(z)$ has a finite deficient value other than a_1, a_2 and a_3. Then $f(z)=P(\exp Az)$ with a quadratic polynomial $P(z)$ and a non-zero constant A.

The object of this paper is to give a further substantial improvement, which gives an essentially sharp form of the above our result.

THEOREM. *Let $f(z)$ be a transcendental entire function which has three distinct finite linearly distributed values c_1, c_2 and c_3. If these three values never lie on any straight line of the complex plane, then*

$$f(z)=P(\exp Az),$$

where $P(z)$ is a quadratic polynomial and A is a non-zero constant.

Considering the sine function or the cosine function, we easily assure that the assumptions of our theorem cannot be improved, in general.

Let $f(z)$ be an entire function having three distinct finite linearly distributed values c_1, c_2 and c_3. By L_j, we denote the straight line on which all the c_j-points of $f(z)$ lie $(j=1, 2, 3)$. With these conventions, we shall show the following four propositions.

PROPOSITION 1. *If at least two of the three lines L_j coincide with each other*

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