I. AMEMIYA, S. OKADA AND Y. OKAZAKI
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PRE-RADON MEASURES ON TOPOLOGICAL SPACES

BY ICHIRO AMEMIYA, SUSUMU OKADA AND YOSHIAKI OKAZAKI

§ 1. Introduction.

There are two directions in the study of the measure theory on arbitrary topological spaces: the theory of Radon measures and the theory of Baire measures. The outline of the developments in these fields is referred to Bourbaki [2], Hirschfeld [8], Schwartz [11] and Varadarajan [13].

The purpose of this paper is to study infinite Borel measures.

Originally, in 1970, the first author has proposed the notion of a pre-Radon measure on a topological space, which is defined as a class of "measures determined by an open base with a smoothness condition" (Amemiya [1]). It seems to be of use for the study of infinite measures, especially Borel measures on a topological space. In this paper, we formulate a pre-Radon measure as a Borel measure (see Definition 2.2) and develop the topics in a survey of Amemiya [1] from a different viewpoint.

Finite pre-Radon measures are said to be τ-smooth Borel measures which have been investigated by many mathematicians. For infinite Borel measures with τ-smoothness, Fremlin [3] recently presented the class of quasi-Radon measures. Our pre-Radon measures are slightly different from quasi-Radon measures.

Main results of this paper are three constructions of pre-Radon measures given in Section 3. The fundamental idea is suggested by Kirk [9]. In Theorem 3.1, we extend a finitely additive set function satisfying some smoothness conditions defined on the ring generated by an open base to a pre-Radon measure. Similarly, in Theorem 3.2 we consider a set function defined on the algebra generated by an open base. In Theorem 3.4, an infinite Baire measure with τ-smoothness on a normal space is extended to a pre-Radon measure. For finite τ-smooth Baire measures, this extension is known (see for example Kirk [9]).

In Section 4, we give the decomposition theorem for σ-finite pre-Radon measures.

In Section 5, we deal with the restriction of pre-Radon measures. We present the several conditions that the restriction is a pre-Radon measure.

In Section 6, we prove the decomposability of pre-Radon measures. For Radon measures, the decomposability is given in [2, § 1, Proposition 9] and for quasi-Radon measures, Fremlin [2, Theorem 72B].

In section 7, we give some topological spaces with the (K)-property (for the

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