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INFINITESIMAL VARIATIONS OF INVARIANT SUBMANIFOLDS OF A KAEHLERIAN MANIFOLD

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§ 0. Introduction.

Recently infinitesimal variations of submanifolds have been studied by Chen [1], Goldstein [2], Ryan [2], Tachibana [3, 4] and one of the present authors [1, 4].

The purpose of the present paper is to study infinitesimal variations of invariant submanifolds of a Kaehlerian manifold and to generalize some of recent results of Tachibana and one of the present authors.

In the preliminary § 1, we state some properties of invariant submanifolds of a Kaehlerian manifold.

In § 2 we prove fundamental formulas in the theory of infinitesimal variations and study complex variations, that is, infinitesimal variations which carry an invariant submanifold into an invariant submanifolds. In § 3, we study holomorphic variations, that is, complex variations which preserve complex structures induced on invariant submanifolds.

In § 4, we study complex conformal variations and prove that a complex conformal variation of a compact invariant submanifold of a Kaehlerian manifold is necessarily isometric and hence holomorphic, (Theorem 4.1). In the last § 5 we prove an integral formula and show some of its applications.

§ 1. Invariant submanifolds of a Kaehlerian manifold.

Let M^{2m} be a real $2m$-dimensional Kaehlerian manifold covered by a system of coordinate neighborhoods $\{U; x^h\}$ and $F_i{}^h$ the almost complex structure tensor and g_{ji} the Hermitian metric tensor, where here and in the sequel, the indices $h, i, j, \cdots$ run over the range $\{1, 2, \cdots, 2m\}$.

Then we have

$$F_i{}^t F_t{}^h = -\delta_i^h, \qquad F_j{}^t F_i{}^s g_{ts} = g_{ji}, \tag{1.1}$$

$$\nabla_j F_i{}^h = 0, \tag{1.2}$$

where ∇_j denotes the operator of covariant differentiation with respect to the Christoffel symbols $\Gamma_j{}^h{}_i$ formed with g_{ji}.

Let M^n be an n-dimensional Riemannian manifold covered by a system of

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