D. E. BLAIR, S. ISHIHARA AND G. D. LUDDEN
KODAI MATH. J.
1 (1978), 75—84

PROJECTABLE ALMOST COMPLEX CONTACT STRUCTURES

BY D. E. BLAIR, S. ISHIHARA AND G.D. LUDDEN

A complex manifold of complex dimension $2m+1$ is said to be a *complex contact manifold* if it admits an open covering $\{u_\alpha\}$ such that on each u_α there is a holomorphic 1-form ω_α with $\omega_\alpha\wedge(d\omega_\alpha)^m\neq 0$ on $u_\alpha\cap u_\beta\neq\emptyset$, $\omega_\beta=f\omega_\alpha$ for some non-vanishing holomorphic function f. In general such a structure is not given by a global 1-form ω; in fact this is the case for a compact complex manifold if and only if its first Chern class vanishes [6]. However, a complex contact manifold is the base space of a principal fibre bundle with 1-dimensional fibres and real contact structure. Homogeneous complex contact manifolds were studied by Boothby in [3].

It is also shown in [6] that the structural group of the tangent bundle of a Hermitian contact manifold M is reducible to $(Sp(m)\cdot Sp(1))\times U(1)$ where $Sp(m)\cdot Sp(1)=Sp(m)\times Sp(1)/\{\pm I\}$ and hence equivalently M admits the following local structure tensors. Let F denote the almost complex structure and g the Hermitian metric on M. Then each coordinate neighborhood admits tensor fields G, H of type (1, 1) and vector fields U, V with covariant forms u and v such that (G, U, V, g) and (H, U, V, g) are metric f-structures with complemented frames (see e.g. [1]), $FU=V$ and $GH=-HG=F+v\otimes U-u\otimes V$. In the overlap of coordinate neighborhoods we have

$$\begin{aligned} G'&=aG+bH, & u'&=au+bv, \\ H'&=-bG+aH, & v'&=-bu+av \end{aligned} \tag{0.1}$$

with $a^2+b^2=1$. Such a structure is called an *almost complex contact structure* [5] and our first project here will be to given an equivalent definition in terms of global tensor fields.

A standard example of a complex contact manifold is the odd-dimensional complex projective space PC^{2m+1}. It is also well known that PC^{2m+1} is a fibre space over the quaternionic projective space PH^m with fibres $S^2\approx PC^1$. In sections 3 and 4 we generalize this situation to a projectable almost complex contact structure on a Kählerian manifold.

§ 1. Almost Complex Contact Structures

In terms of the above local tensor fields G, H, U, V we can define global

Received June 14, 1976