K. YANO
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INFINITESIMAL VARIATIONS OF SUBMANIFOLDS

BY KENTARO YANO

§ 0. Introduction.

The purpose of the present paper is to study variations of the metric tensor, the Christoffel symbols and the second fundamental tensors of submanifolds under infinitesimal variations of the submanifolds.

The method used here is to displace the deformed quantities back parallelly from the displaced point to the original point and to compare the parallelly displaced back quantities and the original quantities, [3], [4].

In § 1, we state formulas for submanifolds of a Riemannian manifold needed for the later discussions including equations of Gauss, Codazzi and Ricci. [1].

In § 2, we consider infinitesimal variations of submanifolds of a Riemannian manifold. We define parallel variations of submanifolds and study their properties.

§ 3 is devoted to the study of variations of the fundamental metric tensor of the submanifold. We discuss isometric, conformal and volume-preserving variations.

We study in § 4 the variations of the Christoffel symbols and those of linear connection induced in the normal bundle. When the submanifold is compact or complete and irreducible, we obtain some global results.

In the last § 5, we study variations of the second fundamental tensors and prove some global propositions. (For normal variations, see [2]).

§ 1. Preliminaries.

Let M^m be an m-dimensional Riemannian manifold covered by a system of coordinate neighborhoods $\{U; x^h\}$ and denote by g_{ji}, Γ^h_{ji}, ∇_j, $K_{kji}{}^h$ and K_{ji} the metric tensor, the Christoffel symbols formed with g_{ji}, the operator of covariant differentiation with respect to Γ^h_{ji}, the curvature tensor and the Ricci tensor of M^m respectively, where and in the sequel the indices $h, i, j, k, \cdots$ run over the range $\{\bar{1}, \bar{2}, \cdots, \bar{m}\}$.

Let M^n be an n-dimensional Riemannian manifold covered by a system of coordinate neighborhoods $\{V; y^a\}$ and denote by g_{cb}, Γ^a_{cb}, ∇_c, $K_{dcb}{}^a$ and K_{cb} the corresponding quantities of M^n respectively, where and in the sequel the indices

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