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Gevrey regularizing effect for a nonlinear Schrödinger equation in one space dimension

By Kazuo TANIGUCHI

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§0 Introduction.

In [3], [1] and [5] we have obtained the Gevrey regularizing effect for the solution of a non-linear Schrödinger equation

$$(1)\qquad \begin{cases} \partial_t u + i\Delta u = f(u), \\ u(0,x) = u_0(x). \end{cases}$$

In these papers, we treat Schrödinger equations whose non-linear terms depend only on the value of the unknown functions. In the present paper, we investigate Gevrey regularizing effect for the equation

$$(2)\qquad \begin{cases} Lu \equiv \partial_t u + i\partial_x^2 u = f(u, \partial_x u), \\ u(0,x) = u_0(x) \end{cases}$$

in one space dimension, whose non-linear term depends also on the derivatives of the unknown functions.

The existence of the solution for the equation (2) are obtained in [2] and [4] in case that $f(u,v)$ is a polynomial in the argument $(u, \bar{u}, v, \bar{v})$. In the general case for $f(u,v)$, however, we can also obtain the existence of the solution as follows:

THEOREM 0. *Assume that a C^∞-function $f(u,v)$ satisfies $\partial_{\bar{v}} f(u,v) = 0$ and $f(0,0) = \partial_v f(0,0) = 0$. Then, for any R_0 there exists a constant $T \equiv T(R_0)$ such that, for any initial data $u_0 \equiv u_0(x)$ with $\|u_0\|_3 \le R_0$ and $\|xu_0\| \le R_0$, the solution $u(t,x)$ of (2) exists in $[0,T]$ and it belongs to $C([0,T]; H^3) \cap C^1([0,T]; H^1)$.*

Our concern is the Gevrey regularizing effect for the solution of (1). So, we omit the proof of Theorem 0 and, in the following, we treat only the Gevrey regularizing effect for the solution (2). The conditions of $f(u,v)$ are the following:

(A.0) $f(u,v)$ is a C^∞ complex valued function in $\boldsymbol{C}^2$, which is holomorphic with respect to v. Moreover, it satisfies $f(0,0) = \partial_v f(0,0) = 0$.

(A.1) Let s satisfy $s \ge 1$. For any positive number K, there exist constants $C = C(K)$ and $A = A(K)$ such that

$$|\partial_u^k \partial_{\bar{u}}^{k'} \partial_v^{k''} f(u,v)| \le CA^{k+k'+k''} k!^s k'!^s k''! \qquad \textit{for } |u|, |v| \le K.$$