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Zariski pairs, fundamental groups and Alexander polynomials

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In this paper we present new examples of Zariski pairs and we compute some invariants of Zariski pairs which were already known. We discuss also the consequences of these new results in the theory of isolated singularities of surface.

We recall the notion of Zariski pair which was introduced in **[A1]**. We will say that two curves C and D are members of a Zariski pair if:

(i) There is a degree-preserving bijection α between the set of irreducible components of C and D and there exist regular neighbourhoods of $T(C)$ and $T(D)$ (of C and D, respectively) such that the pairs $(T(C), C)$ and $(T(D), D)$ are homeomorphic and the homeomorphism respects the bijection above.

(ii) The pairs $(\boldsymbol{P}^2, C)$ and $(\boldsymbol{P}^2, D)$ are not homeomorphic.

We recall that first condition means that there exists also a bijection β between the branches of the singular points of C and D such that:

(i1) If T is a branch at a singular point of C, T and $\beta(T)$ have the same topological type.

(i2) If T, T' are two different branches at singular points of C, then their intersection number equals the intersection number of $\beta(T)$ and $\beta(T')$.

(i3) If T is a branch at a singular point of C and C_T is the irreducible component of C which contains T, then $\alpha(C_T)$ is the irreducible component of D which contains $\beta(T)$.

The first Zariski pair appears in the works of Zariski, see **[Z1]**, **[Z2]**, **[Z3]**: the members of the pair are irreducible sextics with six ordinary cusps; in one case the cusps lie in a conic and they do not in the other one (explicit equations for this case appear in **[O1]** and **[A1]**). Some other examples can be found in **[A1]**. The invariant used to distinguish the members of these pairs is the same: we called it the b-invariant and one construct is as follows:

Let C be a reduced plane curve of degree d and $F(x, y, z) = 0$ a defining equation of C. Let X be any desingularization of the projective hypersurface X_1 in $\boldsymbol{P}^3$ defined by $F(x, y, z) = t^d$; two such desingularizations are birationally equivalent, so the first Betti number of X is an invariant $b(C)$ of the pair $(\boldsymbol{P}^2, C)$. One can take a finer invariant; if one composes the desingularization of X_1 with the restriction to X_1 of the projection in the first three variables x, y, z, then one has a d-sheeted cyclic covering $\sigma: X \to \boldsymbol{P}^2$ which is unramified on $\boldsymbol{P}^2 \backslash C$. The monodromy operator $\tau: X \to X$ acts on $\mathrm{H}^1(X; \boldsymbol{C})$.

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