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Asymptotic behavior of the first exit time of randomly perturbed dynamical systems with a repulsive equilibrium point

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0. Introduction.

Let $X^{\varepsilon}(t,x)$ $(t \geq 0, x \in R^d, \varepsilon > 0)$ be the solution of the following stochastic differential equation:

$$\begin{aligned} &dX^{\varepsilon}(t,x) = b(X^{\varepsilon}(t,x))\,dt + \varepsilon^{1/2}\sigma(X^{\varepsilon}(t,x))\,dW(t), \\ &X^{\varepsilon}(0,x) = x, \end{aligned} \tag{0.1}$$

where $b(\cdot) = (b^i(\cdot))_{i=1}^d : R^d \mapsto R^d$ is Lipschitz continuous, where $\sigma(\cdot) = (\sigma^{ij}(\cdot))_{i,j=1}^d : R^d \mapsto M_d(R)$ is bounded, Lipschitz continuous, and uniformly nondegenerate, and where $W(\cdot)$ is a d-dimensional Wiener process (see [**11**]). $X^{\varepsilon}(t,x)$ can be considered as the small random perturbations of $X^0(t,x)$ for small ε (see [**8**]).

Let $D(\subset R^d)$ be a bounded domain which contains the origin o, with C^2-boundary ∂D, and suppose that $b(x) = o$ if and only if $x = o$. The asymptotic behavior of the first exit time $\tau_D^{\varepsilon}(x)$ of $X^{\varepsilon}(t,x)$ from D defined by

$$\tau_D^{\varepsilon}(x) \equiv \inf\{t > 0; X^{\varepsilon}(t,x) \notin D\} \tag{0.2}$$

has been studied by many authors.

The first result on the asymptotic behavior of $\tau_D^{\varepsilon}(x)$ as $\varepsilon \to 0$ was given by M. I. Freidlin and A. D. Wentzell (see [**5**], [**7**], [**8**], [**18**]).

THEOREM 0.1 ([**8**], p. 127, Theorem 4.2 and [**19**], Lemma 1). *Suppose that* $X^0(t,x) \in D$ $(t > 0)$ *and* $\lim_{t\to\infty} X^0(t,x) = o$ *for all* $x \in \bar{D}$. *Then the following holds; for any* $x \in D$ *and* $\delta > 0$,

$$\begin{aligned} &\lim_{\varepsilon\to 0} P(\exp((V_D - \delta)/\varepsilon) < \tau_D^{\varepsilon}(x) < \exp((V_D + \delta)/\varepsilon)) = 1, \\ &\lim_{\varepsilon\to 0} \varepsilon \log E[\tau_D^{\varepsilon}(x)] = V_D, \end{aligned} \tag{0.3}$$

where we put

$$V_D \equiv \inf\left\{\int_0^t |\sigma(\varphi(s))^{-1}(d\varphi(s)/ds - b(\varphi(s)))|^2\,ds/2; \varphi(0) = o, \varphi(t) \in \partial D, \{\varphi(s); 0 \leq s < t\} \subset D, t > 0\right\}. \tag{0.4}$$