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Solutions of the Dirichlet problem on a cone with continuous data

Dedicated to Professor Yasuo Okuyama on his 60th birthday

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1. Introduction

Let $\boldsymbol{R}$ and $\boldsymbol{R}_+$ be the set of all real numbers and all positive real numbers, respectively. The boundary and the closure of a set S in the n-dimensional Euclidean space $\boldsymbol{R}^n (n \geq 2)$ are denoted by ∂S and $\bar{S}$, respectively. We also introduce the spherical coordinates $(r, \Theta), \Theta = (\theta_1, \theta_2, \dots, \theta_{n-1})$, in $\boldsymbol{R}^n$ which are related to the cartesian coordinates $(X, y), X = (x_1, x_2, \dots, x_{n-1})$ by the formulas

$$x_1 = r\left(\prod_{j=1}^{n-1} \sin\theta_j\right) \quad (n \geq 2), \quad y = r\cos\theta_1,$$

and if $n \geq 3$,

$$x_{n+1-k} = r\left(\prod_{j=1}^{k-1} \sin\theta_j\right)\cos\theta_k \quad (2 \leq k \leq n-1),$$

where

$$0 \leq r < \infty, 0 \leq \theta_j \leq \pi (1 \leq j \leq n-2; n \geq 3), \quad -2^{-1}\pi < \theta_{n-1} \leq 2^{-1}3\pi.$$

The unit sphere (the unit circle, if $n = 2$) and the upper half unit sphere $\{(1, \theta_1, \theta_2, \dots, \theta_{n-1}) \in \boldsymbol{R}^n; 0 \leq \theta_1 < \pi/2\}$ (the upper half unit circle $\{(1, \theta_1) \in \boldsymbol{R}^2; -\pi/2 < \theta_1 < \pi/2\}$, if $n = 2$) in $\boldsymbol{R}^n$ are denoted by $\boldsymbol{S}^{n-1}$ and $\boldsymbol{S}_+^{n-1}$, respectively. The half-space (the half-plane, if $n = 2$)

$$\{(X, y) \in \boldsymbol{R}^n; X \in \boldsymbol{R}^{n-1}, y > 0\} = \{(r, \Theta) \in \boldsymbol{R}^n; \Theta \in \boldsymbol{S}_+^{n-1}, 0 < r < \infty\}$$

is denoted by $\boldsymbol{T}_n$.

Given a domain $D \subset \boldsymbol{R}^n$ and a continuous function g on ∂D, we say that h is a solution of the (classical) Dirichlet problem on D with g, if h is harmonic in D and

$$\lim_{P \in D, P \to Q} h(P) = g(Q)$$

for every $Q \in \partial D$. If D is a smooth bounded domain, then the existence of a solution of the Dirichlet problem and its uniqueness is completely known (see e.g. [**11**, Theorem 5.21]). When D is the typical unbounded domain $\boldsymbol{T}_n$, Helms [**13**, p.42 and p.158] states that even if $g(x)$ is a bounded continuous function on $\partial\boldsymbol{T}_n$, the solution of the Dirichlet