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# On the classification of the third reduction with a spectral value condition

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## 0. Introduction.

This paper is a continuation of [**BS1**] and [**BS2**], and gives a new method to investigate the structure of the third reduction, by adjusting the spectral value (see Definition 1.7).

The first and second reductions (see Definitions 2.1) are studied in [**BS1**] and [**Fj2**]. When we face the third reduction, or equivalently, consider a reduction series $X_0 \xrightarrow{\phi_1} X_1 \xrightarrow{\phi_2} X_2 \xrightarrow{\phi_3} X_3$, severe difficulties are caused by

(1) the isolated, 2-factorial, terminal singularities of $X_2$,
(2) the ray contractions of flipping type, through which the third reduction $\phi_3$ factors, and
(3) that $L_2$ (see Definitions 2.1) is not necessarily ample, spanned or even a line bundle.

However if we put some condition on the spectral value $u(X_0, L_0)$, we can exclude these bad situations. More precisely, we will use the main theorem, [**BS2,** Theorem (3.1.4)], which says that if the *third spectral condition*: $u(X_0, L_0) > 2\lceil (n-1)/3 \rceil$ is satisfied, then $\phi_1$ and $\phi_2$ are isomorphisms. Hence $X_2$ is smooth, and $L_2$ is a very ample line bundle, which settle (1) and (3). At the same time, this condition kills the ray contractions of flipping type in (2). By means of it, we will have Propositions 2.5, 2.7 which express nice properties of the third reductions chosen by this condition. The third reduction $\phi_3: X_2 \rightarrow X_3$ with the above third spectral condition is called the *spectral third reduction* and is denoted by $\phi: X \rightarrow Y$ (see Definition 2.2). Applying [**Na1,** Theorem 1.3, Propositions 2.1, 2.3], the classification of the third adjoint contractions, we will obtain the structure theorem on the positive dimensional fibers of the spectral third reduction, the main Theorem 2.3, where it will also be shown that $Y$ has factorial, terminal singularities.

To classify the next reduction by the same method, it is important to ask what spectral condition implies that the third reduction is trivial. This condition, the fourth spectral condition, is given in Definition 3.1. To prove this,