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Self maps of spaces

Dedicated to Professor Tsuyoshi Watabe on his sixtieth birthday

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1. Introduction and statements of results.

Given a path-connected space X, we write

$$QH^{n}(X;\boldsymbol{K})=\tilde{H}^{n}(X;\boldsymbol{K})/\left\{\sum_{i}\tilde{H}^{i}(X;\boldsymbol{K})\cdot\tilde{H}^{n-i}(X;\boldsymbol{K})\right\}$$

for $\boldsymbol{K}=\boldsymbol{Z}, \boldsymbol{Q}$.

If G is a connected Lie group, then the k-fold product ${}^{k}id$ of the identity map of G satisfies $({}^{k}id)^{*}(x)=kx$ for all $x\in QH^{*}(G;\boldsymbol{Q})$. This property was important in [**5**]. Apart from extending Haibao's results on H-spaces to more general spaces, the following problem seems interesting in its own sense.

PROBLEM. *Given a function* $\theta:\{1,2,\cdots\}\rightarrow\boldsymbol{Z}$, *is there a self map* μ_{θ} *of* X *such that*

(1.1) $\quad \mu_{\theta}^{*}(x)=\theta(\deg(x))x$ *for all homogeneous elements* $x\in QH^{*}(X;\boldsymbol{Q})$?

DEFINITION. We call a path-connected space X an M_{θ}-space if it has a self map μ_{θ}, which is called an M_{θ}-structure on X, satisfying (1.1).

When θ is the constant function to $k\in\boldsymbol{Z}$, we denote M_{θ} and μ_{θ} by M_{k} and μ_{k}, respectively. When there exist an integer k and a function $e:\{1,2,\cdots\}\rightarrow\{0,1,2,\cdots\}$ with $\theta(n)=k^{e(n)}$ for all $n\geqq 1$, we denote M_{θ} and μ_{θ} by $M_{k^{e}}$ and $\mu_{k^{e}}$, respectively. Note that every path-connected space is an M_{0} and M_{1} space.

We shall need some finiteness condition on X. That is, we will frequently assume some of the following:

(1.2) $\quad H_{n}(X;\boldsymbol{Z})$ is finitely generated for all n;

(1.3) $\quad \dim H_{n}(X;\boldsymbol{Q})<\infty$ for all n;

(1.4) $\quad \dim H^{*}(X;\boldsymbol{Q})<\infty$;

(1.5) $\quad \dim QH^{*}(X;\boldsymbol{Q})<\infty$;