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The time slicing approximation of the fundamental solution for the Schrödinger equation with electromagnetic fields

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1. Introduction.

In this paper we construct the fundamental solution of the Schrödinger equation for a charged particle in electromagnetic fields:

$$i\hbar\frac{\partial}{\partial t}u=\left[\frac{1}{2}\sum_{k=1}^{d}\left(-i\hbar\frac{\partial}{\partial x_k}-A_k(t,x)\right)^2+V(x)\right]u$$
$$=H(\hbar,t)u\,,\quad t\in\boldsymbol{R},\ x=(x_1,\cdots,x_d)\in\boldsymbol{R}^d,\tag{1.1}$$

where $0<\hbar\leqq 1$ is a parameter and $A(t,x)=(A_1(t,x),\cdots,A_d(t,x))$ and $V(x)$ are the vector and scalar potentials of the fields which satisfy the following assumption:

ASSUMPTION (A). For $k=1,\cdots,d$, $A_k(t,x)$ is a real-valued function of $(t,x)\in\boldsymbol{R}\times\boldsymbol{R}^d$, and for any multi-index α, $\partial_x^\alpha A_k(t,x)$ is C^1 in $(t,x)\in\boldsymbol{R}\times\boldsymbol{R}^d$. There exists $\varepsilon>0$ such that

$$|\partial_x^\alpha A_k(t,x)|+|\partial_x^\alpha\partial_t A_k(t,x)|\leqq C_\alpha,\quad|\alpha|\geqq 1,\ (t,x)\in\boldsymbol{R}\times\boldsymbol{R}^d,\tag{1.2}$$

$$|\partial_x^\alpha B(t,x)|\leqq C_\alpha(1+|x|)^{-1-\varepsilon},\quad|\alpha|\geqq 1,\tag{1.3}$$

where $B(t,x)$ is a skew symmetric matrix with (k,l)-component $B_{kl}(t,x)=(\partial A_l/\partial x_k-\partial A_k/\partial x_l)(t,x)$ and $|B|$ denotes the norm of matrix B regarded as an operator on $\boldsymbol{R}^d$. $V(x)$ is a real-valued C^∞ function which satisfies

$$|\partial_x^\alpha V(x)|\leqq C_\alpha,\quad|\alpha|\geqq 2.\tag{1.4}$$

Let $q(\tau)$ be a classical path in the electromagnetic field joining $y\in\boldsymbol{R}^d$ at time s to $x\in\boldsymbol{R}^d$ at time t: $q(\tau)$ satisfies Lagrange's equation

$$\dot{q}(\tau)=v(\tau),\quad\dot{v}(\tau)=B(\tau,q(\tau))v(\tau)+F(\tau,q(\tau)),\quad s\leqq\tau\leqq t,\tag{1.5}$$

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