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Polarized threefolds with non-zero effective anti-adjoint divisors

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Introduction.

In this paper we study the structure of a polarized threefold (Y, L) such that Y is a normal projective threefold with at worst $\boldsymbol{Q}$-factorial terminal singularities and that its anti-adjoint divisor $-(K_Y+L)$ is linearly equivalent to a non-zero effective Weil divisor. For a polarized variety, the property of its adjoint divisor has been studied by many algebraic geometers (see e.g., T. Fujita [**F**] Chapter II). In principle, the adjoint divisor has positivity properties (ampleness, spannedness, nefness and so on) except a few cases in which (Y, L) has some special structure (see e.g., loc. cit.). Our assumption means that the adjoint bundle is far from being positive, so we expect that the structure of such polarized threeholds can be investigated well.

The other motivation is to study non-normal Gorenstein Fano threefolds. M. Reid classified non-normal Gorenstein Del Pezzo surfaces in [**R**] via the minimal resolutions of their normalizations. Let X be a non-normal Gorenstein Del Pezzo surface, Y the normalization of X and L the ample Cartier divisor corresponding to the inverse image of the anti-dualizing sheaf ω_X^{-1}. By using the adjunction formula he showed that there exists a non-zero effective Weil divisor Δ such that $-K_Y \sim L+\Delta$. After the process above he took the minimal resolution of the normalization Y, and classified the all possibilities of it.

Now we consider the case of Gorenstein Fano threefolds. Let X be a Gorenstein Fano threefold, Y the normalization of X, and L the ample Cartier divisor on Y such that the invertible sheaf $\mathcal{O}_Y(L)$ is isomorphic to the inverse image of the anti-dualizing sheaf ω_X^{-1}. Then there exists a non-zero effective Weil divisor Δ such that $-K_Y \sim L+\Delta$ by using adjunction theory as before. Now we want to apply minimal model program. For this purpose we assume that Y has at worst $\boldsymbol{Q}$-factorial terminal singularities (if Y has worse singularities, minimal model program is difficult to apply). Such a pair (Y, L) is a sort of objects which we will study in this paper.

In section 1, we prepare several results on projective surfaces for the later use. The results and their proofs are very similar to the ones treated by M.