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Ineffability and partition property on $\mathcal{P}_\kappa\lambda$

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1. Introduction.

Magidor [11] proved that if part*(κ, λ) holds, then κ is λ-ineffable. Abe [1] proved that the reverse implication also holds under the assumption of that λ is ineffable. In this paper, we shall prove the following two theorems.

THEOREM 1. *If κ is completely $\lambda^{<\kappa}$-ineffable, then* part*(κ, $\lambda^{<\kappa}$) *holds.*

THEOREM 2. *Assume that there exists an $\alpha<\kappa$ such that $2^\delta\leqq\delta^{+\alpha}$ for all $\delta<\kappa$. Then, if κ is $\lambda^{<\kappa}$-ineffable, then* part*(κ, $\lambda^{<\kappa}$) *holds.*

In order to prove Theorem 1, we need to study a hierarchy of ideals which are associated with partition property and ineffability, and the correspondence between $\mathcal{P}_\kappa\lambda$ and $\mathcal{P}_\kappa\lambda^{<\kappa}$.

The hierarchy of ideals will be dealt in sections 4 and 5 and the correspondence in section 6. The theorems will be proved in section 7.

2. Notation and basic facts.

Throughout this paper, κ denotes a regular uncountable cardinal. Let $\mathcal{I}$ be an ideal on a set S. $\mathcal{I}^*$ denotes the dual filter of $\mathcal{I}$ and $\mathcal{I}^+$ the set $\mathcal{P}(S)\setminus\mathcal{I}$. A subset W of $\mathcal{I}^+$ is *$\mathcal{I}$-disjoint,* if $X\cap Y\in\mathcal{I}$ for all distinct $X, Y\in W$. An $\subset$-maximal $\mathcal{I}$-disjoint subset is called an *$\mathcal{I}$-partition.* For any set $X\subset S$, $\mathcal{I}^+|X$ denotes $\mathcal{I}^+\cap\mathcal{P}(X)$. For any $f: S\to T$, $f_*(\mathcal{I})$ denotes the ideal $\{Y\subset T\,|\,f^{-1}Y\in\mathcal{I}\}$ on T.

Let A be a set such that $\kappa\subset A$. $\mathcal{P}_\kappa A$ denotes the set $\{x\subset A\,|\,|x|<\kappa\}$. Let Y be a subset of $\mathcal{P}_\kappa A$. $[Y]^2$ denotes the set $\{(x, y)\in Y^2\,|\,x\subset y$ and $x\neq y\}$. For any function $F: [Y]^2\to 2$, a subset H of Y is said to be *homogeneous for F,* if $|F''[H]^2|\leqq 1$. For any $B\supset A$, the function $p: \mathcal{P}_\kappa B\to\mathcal{P}_\kappa A$ which is defined by $p(y)=y\cap A$ is called *the projection.* For each $x\in\mathcal{P}_\kappa A$, $\hat{x}$ denotes the set $\{y\in\mathcal{P}_\kappa A\,|\,x\subset y$ and $x\neq y\}$ and Q_x the set $\{t\subset x\,|\,|t|<|x\cap\kappa|\}$. $I_{\kappa,A}$ denotes the ideal $\{X\subset\mathcal{P}_\kappa A\,|\,X\cap\hat{y}=\emptyset$, for some $y\in\mathcal{P}_\kappa A\}$. An element of $I_{\kappa,A}^+$ is called *unbounded.* A subset of $\mathcal{P}_\kappa A$ is called *club,* if it is unbounded and closed under unions of $\subset$-increasing chains with length $<\kappa$. A subset X of $\mathcal{P}_\kappa A$ is called