J. Math. Soc. Japan
Vol. 49, No. 1, 1997

Generalized #-unknotting operations

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(Received Jan. 12, 1995)

Introduction.

We shall work in the P.L. and locally flat category. We discuss oriented knots and links in S^3. Two knots are equivalent if there is an ambient isotopy of S^3 carrying one knot to the other.

H. Murakami [6] showed that any knot can be changed into a trivial knot by repeatedly altering a diagram of the knot as in Figure 0.

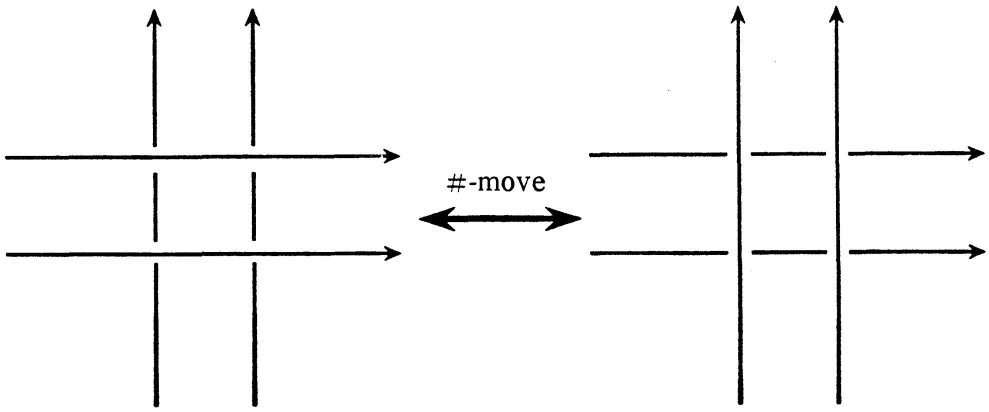

Figure 0.

This move on a diagram is called the #-move or the #-unknotting operation. In this note, generalizing this, we define for any prime p, a $\#^{p}$*-move* on a knot diagram as shown in Figure 1. Note that even if p is fixed, x and y in Figure 1 may vary. (It is easy to define $\#^{p}$-moves for any integers p. However, if p' is a factor of p, then a $\#^{p}$-move is also a $\#^{p'}$-move. We thus consider $\#^{p}$-moves only for prime numbers p.) The #-unknotting operation and the pass-move [4] are examples of $\#^{2}$-moves.

We shall show that for any prime p any knot can be transformed into a trivial knot by a finite sequence of $\#^{p}$-moves (Theorem 1.1). (In fact, if p is odd, a combination of a certain $\#^{p}$-move and Reidemeister moves achieves a crossing change.) Then we can define the $\#^{p}$-unknotting number $u^{p}(K)$ much like the ordinary unknotting number. Since a family of $\#^{p}$-moves is a wide variety of diagramatic changes, one might initially think that every knot can be untied