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On morphisms into contractible surfaces of Kodaira logarithmic dimension 1

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1. Introduction.

The Abhyankar-Moh-Suzuki theorem [**AM**], [**Su**] and the Lin-Zaidenberg theorem [**LZ**] give a complete classification of polynomial injection of the complex line $\boldsymbol{C}$ into the complex plane $\boldsymbol{C}^2$. There is also classification of morphisms of $\boldsymbol{C}$ into $\boldsymbol{C}^2$ for which the only singular point of the image is a node [**N**]. Since smooth contractible complex algebraic surfaces have a lot in common with the plane, it is interesting to consider similar questions for them. There are some results in this direction which we remind now. It is known that every smooth contractible complex algebraic surface has Kodaira logarithmic dimension either 1 or 2 [**GM**], [**F**] unless this surface is isomorphic to $\boldsymbol{C}^2$. Zaidenberg proved that there is no polynomial injection of $\boldsymbol{C}$ into a smooth contractible surface of Kodaira logarithmic dimension 2 [**Z**]. But it is not so for contractible surfaces with Kodaira logarithmic dimension 1. Every of these surfaces contains a curve isomorphic to $\boldsymbol{C}$ [**Z**]. Miyanishi, Sugie and Tsunoda [**MS**], [**MT**] reproved the result of Zaidenberg in the case Kodaira logarithmic dimension 2, and in the case of Kodaira logarithmic dimension 1 they showed that there exists only one contractible curve which, of course, coincides with the curve we mentioned above. (They do not make the assumption about the smoothness of this curve).

In this paper we study contractible surfaces of Kodaira logarithmic dimension $\bar{k}=1$ only. We obtain a powerful generalization of the result of Zaidenberg, Miyanishi and Sugie. Namely we described all morphisms (not necessarily injections) from $\boldsymbol{C}$ into such surfaces. Our classification is unexpectedly simple due to the result of Petrie and tom Dieck who represent some of contractible surfaces with $\bar{k}=1$ as hypersurface in $\boldsymbol{C}^3$ [**PtD**]. Moreover, using the same approach we obtain a complete classification of morphisms from a once-punctured Riemann surface into a contractible surface W with $\bar{k}(W)=1$ (section 3). From this result follows that the Abhyankar-Singh property [**AS**] holds for W, i.e., if f is a regular function on W whose zero fiber is isomorphic to a once-punctured Riemann surface then every fiber of f has one puncture only.