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Topological Anosov maps of infra-nil-manifolds

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§ 0. Introduction.

We shall discuss a part of a problem of whether the universal model of Anosov diffeomorphisms exists. Concerning with this problem Manning [**Ma2**] proved that every Anosov diffeomorphism of an infra-nil-manifold is topologically conjugate to a hyperbolic infra-nil-automorphism. From the remarkable proof of his result and the work of Franks [**Fr**], Aoki and Hiraide has been studied the dynamics of covering maps of a torus ([**Ao-Hi**]).

We shall show in this paper that some of the results stated in [**Ao-Hi**] become realistic for infra-nil-manifolds as follows.

THEOREM 1. *Let $f: N/\Gamma \to N/\Gamma$ be a covering map of an infra-nil-manifold and denote as $A: N/\Gamma \to N/\Gamma$ the infra-nil-endomorphism homotopic to f.*

If f is a TA-map, then A is hyperbolic and the inverse limit system of $(N/\Gamma, f)$ is topologically conjugate to the inverse limit system of $(N/\Gamma, A)$.

THEOREM 2. *Let f and A be as in Theorem* 1. *Then the following statements hold*:

(1) *if f is a TA-homeomorphism, then A is a hyperbolic infra-nil-automorphism and f is topologically conjugate to A,*

(2) *if f is a topological expanding map, then A is an expanding infra-nil-endomorphism and f is topologically conjugate to A.*

In the statement of Theorem 2 it notices that (1) is a generalization of Manning [**Ma2**].

First we shall explain here the definitions and notations used above. Let X and Y be compact metric spaces and let $f: X \to X$ and $g: Y \to Y$ be continuous surjections. Then f is said to be *topologically conjugate* to g if there exists a homeomorphism $\varphi: Y \to X$ such that $f \circ \varphi = \varphi \circ g$.

Let X be a compact metric space with metric d. For $f: X \to X$ a continuous surjection, we let

$$X_f = \{(x_i) : x_i \in X \text{ and } f(x_i) = x_{i+1},\ i \in \mathbf{Z}\},$$

$$\sigma_f((x_i)) = (f(x_i)).$$