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On Thom polynomials of the singularities D_k and E_k

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Introduction.

Let A_k, D_k and E_k denote the types of the singularities of function germs studied in [**4**]. Let N and P denote smooth manifolds. When a C^∞ stable map germ $f:(N, x)\rightarrow(P, y)$ is C^∞ equivalent to a versal unfolding of a function germ with singularity A_k, D_k or E_k, we say that f has a singularity of type A_k, D_k or E_k at x respectively (see, for example, their normal forms of [**2**, Section 1]). When every singularity of a smooth map f is of type A_k or D_k (resp. A_k, D_k or E_k) with any number k, we say that f is AD-regular (resp. ADE-regular) in this paper.

Let X_k be one of A_k, D_k or E_k. We define $S_{\bar{X}_k}(f)$ to be the topological closure of the subset $S_{X_k}(f)$ consisting of all singular points of type X_k of f. We can consider the fundamental class of $S_{\bar{X}_k}(f)$ in $H_*(N; Z/2Z)$ and define the Thom polynomial of X_k for f as its Poincaré dual class denoted by $P(X_k, f)$. As usual we expect that it is represented by Stiefel-Whitney classes $w_j(TN-f^*(TP))$ (cf. [**6**]).

The purpose of this paper is to give formula calculating $P(D_k, f)$ for AD-regular maps and $P(E_k, f)$ for ADE-regular maps in a finite process ([Theorems 4.1 and 4.2]). This kind of formulas first appeared in [**9**] and [**10**] to calculate Thom polynomials of the singularities of type Σ^i and $\Sigma^{i,j}$. Their results are reviewed in Section 1. In our case of $X_k=D_k$ or E_k, we have the submanifolds $\sum X_k$ constructed in the infinite jet space $J^\infty(N, P)$ in [**2**] such that if the jet extension $j^\infty f$ of f is transverse to $\sum X_k$, then we have $S_{X_k}(f)=(j^\infty f)^{-1}(\sum X_k)$. Using the properties of $\sum X_k$ in $J^\infty(N, P)$ reviewed in Section 2, we lift $S_{\bar{X}_k}(f)$ up to a submanifold S of the total space of a certain flag bundle over N in Sections 5 and 6 so that the Poincaré dual class of S is the Euler class of some vector bundle over this total space related to the normal bundle of $\sum X_k$. This means that $P(X_k, f)$ is the image of this Euler class by the Gysin homomorphism of this flag bundle. For singularities A_k, see the similar result of [**1**].

In Section 7 we see that Theorems 4.1 and 4.2 are generalized to the situations of smooth maps into foliated manifolds or of smooth sections of fibre