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Local cohomology modules of indecomposable surjective-Buchsbaum modules over Gorenstein local rings

By Takesi KAWASAKI[1)]

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1. Introduction.

Let A be a Noetherian local ring with maximal ideal $\mathfrak{m}$. We assume that $\dim A=d>0$. The local cohomology functor $H_{\mathfrak{m}}^{i}(-)$ was defined by Grothendieck [**11**] and he showed that for any finitely generated A-module M, the i-th local cohomology module $H_{\mathfrak{m}}^{i}(M)$ vanishes unless

$$\operatorname{depth} M \leqq i \leqq \dim_{A} M$$

and that $H_{\mathfrak{m}}^{i}(M)\neq 0$ if $i=\operatorname{depth} M$ or $i=\dim_{A}M$. We refer to the local cohomology modules $H_{\mathfrak{m}}^{i}(M)$ for which $\operatorname{depth} M<i<\dim_{A}M$ as the intermediate local cohomology modules of M. Pathological behaviors of intermediate local cohomology modules for general Noetherian local rings were reported by several authors. Firstly Sharp [**20**] gave examples of Noetherian local rings whose intermediate local cohomology modules either all vanish or are all non-zero. Furthermore Evans and Griffith [**7**] gave a Noetherian local ring with prescribed local cohomology modules, that is, let $d\geqq 2$ and $h_{1}, \cdots, h_{d-1}\geqq 0$ be arbitrary integers. Then there is a Noetherian local domain A of dimension d such that

$$l_{A}(H_{\mathfrak{m}}^{i}(A))=h_{i} \quad \text{for all } 1 \leqq i \leqq d-1.$$

By modifying their argument, Goto [8] obtained such a ring from among Buchsbaum local rings. Here a finitely generated A-module M is said to be Buchsbaum if the difference $l_{A}(M/\mathfrak{q}M)-e_{\mathfrak{q}}(M)$ is an invariant of M not depending on the choice of the parameter ideal $\mathfrak{q}$ for M. Moreover a Noetherian local ring A is said to be Buchsbaum if it is a Buchsbaum module over itself.

In this paper we are interested in behaviors of local cohomology modules of *finitely generated indecomposable modules*. Goto [**9**] gave a structure theorem for maximal Buchsbaum modules over regular local rings, that is, if A is a regular local ring of dimension $d>0$ and M is an indecomposable maximal

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