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Non smooth Lagrangian sets and estimations of micro-support

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1. Notation and review.

Let X be a real C^1 manifold and let $Y \subset X$ be a closed submanifold. One denotes by $\pi: T^*X \to X$ the cotangent bundle to X and by T_Y^*X the conormal bundle to Y in X.

One denotes by $\mathrm{D}^b(X)$ the derived category of the category of bounded complexes of sheaves of $\boldsymbol{C}$-vector spaces on X. For F an object of $\mathrm{D}^b(X)$, one denotes by $\mathrm{SS}(F)$ its micro-support, a closed, conic, involutive subset of T^*X.

Let $A \subset X$ be a closed C^1-convex subset at $x_0 \in A$ (i.e., A is convex for a choice of local C^1 coordinates at x_0). One denotes by $\boldsymbol{C}_A$ the sheaf which is zero on $X \setminus A$ and the constant sheaf with fiber $\boldsymbol{C}$ on A. In order to describe $\mathrm{SS}(\boldsymbol{C}_A)$ fix a local system of coordinates $(x)=(x', x'')$ at x_0 so that A is convex and $Y=\{x \in X; x''=0\}$ is its linear hull. Denote by $j: Y \to X$ the embedding and by ${}^t j': Y \times_X T^*X \to T^*Y$ the associated projection. One has

$$\mathrm{SS}(\boldsymbol{C}_A) = {}^t j'(N_Y^*(A)),$$

where $N_Y^*(A)$ denotes the conormal cone to A in Y. In other words, $(x; \xi) \in \mathrm{SS}(\boldsymbol{C}_A)$ if and only if $x \in A$ and the half space $\{y \in X; \langle y-x, \xi \rangle \geqq 0\}$ contains A. By analogy with the smooth case, we set $T_A^*X = \mathrm{SS}(\boldsymbol{C}_A)$.

For $p \in T^*X$, $\mathrm{D}^b(X; p)$ denotes the localization of $\mathrm{D}^b(X)$ with respect to the null system $\{F \in \mathrm{D}^b(X); p \notin \mathrm{SS}(F)\}$. One also considers the microlocalization bifunctor $\mu hom(\cdot, \cdot)$ which is defined in [**K-S**].

REMARK 1.1. In [**K-S**] the bifunctor μhom is considered only for C^2 manifolds but it is clear that its definition is possible for a C^1 manifold as well. Roughly speaking, this functor is the composition of the specialization functor (which is defined as long as the normal deformation is defined, i.e., for C^1 manifolds) and the Fourier-Sato transform which is defined for vector bundles over any locally compact space.

If X is of class C^2 one has the following estimate: