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Hausdorff dimension of Markov invariant sets

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Introduction.

One of the old questions about exceptional minimal sets of codimension-one C^2-foliations of compact manifolds reads (compare [**La**]): Is the Lebesgue measure $|\mathcal{M}|$ of any exceptional minimal set $\mathcal{M}$ equal to 0? The answer in general is still unknown. The class of Markov minimal sets was introduced by John Cantwell and Lawrence Conlon [**CC**] in the context of this question. Among the other results, they proved that $|\mathcal{M}|=0$ if $\mathcal{M}$ is a Markov exceptional minimal set. The same result in the particular case of a Markov exceptional minimal set with holonomy generated by two maps defined on a common interval was obtained in [**Mat**].

In [**LaW**], while studying relations between different invariants describing the dynamics of foliations, the authors observed that the question about the Hausdorff dimension $\dim_H$ of exceptional minimal sets is also of some interest. Since the inequality

$$(1) \qquad \dim_H(\mathcal{M}) < \dim M,$$

M being the foliated manifold, implies that $|\mathcal{M}|=0$, Markov exceptional minimal sets seem to be good candidates to satisfy (1). In fact, this is our result here.

THEOREM. *If $\mathcal{M}$ is a Markov exceptional minimal set of a codimension-one C^2-foliation $\mathcal{F}$ of a compact manifold M, then $\mathcal{M}$ satisfies inequality* (1).

The Theorem follows immediately from the description of Markov exceptional minimal sets given in [**CC**] and the following.

PROPOSITION. *If Γ is a finitely generated Markov pseudogroup of local C^2-diffeomorphisms of the real line $\boldsymbol{R}$ and Z_0 is its Markov invariant set, then*

$$(1) \qquad \dim_H(Z_0) < 1.$$

The idea of the proof of the Proposition is very similar to that of Theorem 3 in [**CC**]. We use several preparatory Lemmas of [**CC**] as well as some subtle estimates of [**Mat**]. However, we believe that the result itself as well as