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Boundary distance functions and q-convexity of pseudoconvex domains of general order in Kähler manifolds

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Introduction.

Let M be an n-dimensional Kähler manifold with C^∞ Kähler metric G, let D be an open subset of M, and let $d_{\partial D}$ be the boundary distance function of D induced by the metric G.

When D is pseudoconvex (in the usual sense) in M, the plurisubharmonicity of the function $-\log d_{\partial D}$ is closely related to the holomorphic bisectional curvature of M. Takeuchi [**26**] first showed that, if D is a pseudoconvex open subset of the complex projective space $P^n(\boldsymbol{C})$ and if $d_{\partial D}$ is the boundary distance function of D with respect to the Fubini-Study metric on $P^n(\boldsymbol{C})$, the function $-\log d_{\partial D}$ is strongly plurisubharmonic on D. After the works of Takeuchi [**27**], Elencwajg [**6**], Suzuki [**24**] and others, Greene-Wu [**11**] differential-geometrically gave an estimate from below for 'the modulus of plurisubharmonicity' of the function $-\log d_{\partial D}$, and showed that a relatively compact, pseudoconvex open subset D of M is 1-complete (and hence Stein) if M has positive holomorphic bisectional curvature.

In this paper, we shall extend the result to the case where D is pseudoconvex of order $n-q$ in M and show that D is q-convex or q-complete (with corners) in several cases.

An open subset D of M is said to be pseudoconvex of order $n-q$, $1\leqq q\leqq n$, in M if, roughly speaking, the complement $M\setminus D$ has the same continuity as an analytic set of pure dimension $n-q$. Pseudoconvex open subsets in the usual sense are pseudoconvex of order $n-1$. If $D\subset M$ is weakly q-convex, then D is pseudoconvex of order $n-q$ in M. However, when $2\leqq q\leqq n-1$, the converse is not valid even if $D\subset\boldsymbol{C}^n$ (see Diederich-Fornaess [**4**] and Matsumoto [**13**]). By Fujita [**8**], an open subset D of $\boldsymbol{C}^n$ is pseudoconvex of order $n-q$ in $\boldsymbol{C}^n$, if and only if D has an exhaustion function which is pseudoconvex of order $n-q$ on D. Therefore, by the approximation theorem of Bungart [**3**], an open subset D of M is pseudoconvex of order $n-q$ in M, if and only if D is locally q-complete with corners in M in the sense of Peternell [**16**] (for the precise, see §§ 1 and 2).