J. Math. Soc. Japan
Vol. 47, No. 4, 1995

On dimensions of non-Hausdorff sets for plane homeomorphisms

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(Received Jan. 27, 1994)

§ 1. Introduction.

A homeomorphism is called *flowable* if there exists a topological flow whose time one map is that homeomorphism. An orientation preserving fixed point free homeomorphism of $\boldsymbol{R}^2$ which is not flowable was constructed by Kerékjártó in 1934 ([**9**]). In order to show the homeomorphism is not flowable, he defined "singular points", at which the family $\{f^n\}_{n\in Z}$ is not equicontinuous with respect to the elliptic metric.

The set of "singular points" coincides with the following non-Hausdorff set (see [**10**], [**11**]): Let f be an orientation preserving fixed point free homeomorphism of $\boldsymbol{R}^2$. Denote by $\pi:\boldsymbol{R}^2\to\boldsymbol{R}^2/f$ the quotient map which maps each orbit of f to a point. Then $\boldsymbol{R}^2/f$ is a non-Hausdorff manifold because the non-wandering set of f is empty ([**1**], [**5**] Corollary 2.3). A point p of $\boldsymbol{R}^2$ is called *non-Hausdorff* if $\pi(p)$ is not "Hausdorff" in $\boldsymbol{R}^2/f$. We call the set of all non-Hausdorff points the *non-Hausdorff set*, denoted by $NH(f)$.

In this paper, we characterize $NH(f)$ by the limit set of continua and give the dimension of $NH(f)$.

MAIN THEOREM. *Let f be an orientation preserving fixed point free homeomorphism of $\boldsymbol{R}^2$. Then $NH(f)$ is one-dimensional unless it is empty.*

In the following, we assume that all homeomorphisms of $\boldsymbol{R}^2$ are orientation preserving and without fixed points, and the topology of $\boldsymbol{R}^2$ is given by the Euclidean metric.

In § 2, we give a precise definition of non-Hausdorff points and characterize $NH(f)$ by the limit sets of continua (Theorem 1). The main theorem is proved in § 3 by using Theorem 2 in § 2 and Theorem 3 in § 3.

The author wishes to thank Professor T. Tsuboi, S. Matsumoto, T. Inaba, L. Guillou and A. Marin for their helpful suggestions and encouragement.