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Expansiveness of homeomorphisms and dimension

Dedicated to Professor Akihiro Okuyama on his 60th birthday

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0. Introduction.

In [8], Mañé proved that if a compact metric space X admits an expansive homeomorphism, then X is finite dimensional. In [6, 7], we introduced the notion of continuum-wise (fully) expansive homeomorphism and investigated the several properties. The class of continuum-wise expansive homeomorphisms is much larger than the class of expansive homeomorphisms. In relation to dimension theory, the following results were proved; (1) if a compact metric space X admits a continuum-wise expansive homeomorphism, then X is finite dimensional [6], and (2) if a continuum X admits a positively continuum-wise fully expansive map, then X is 1-dimensional [7]. In this paper, we define the notion of barriers of a homeomorphism $f: X \to X$ and an index $B(f)$. We are interested in the relation between the index $B(f)$ and the dimension of X. The following theorem is proved; if $f: X \to X$ is a continuum-wise expansive homeomorphism of a compact metric space X, then $\dim X \leqq B(f) \leqq 2 \cdot \dim X < \infty$. As a corollary, if $f: X \to X$ is a continuum-wise fully expansive homeomorphism, then $\dim X \leqq B(f) \leqq 2$.

1. Definitions and preliminaries.

By a *continuum*, we mean a compact metric connected nondegenerate space. A homeomorphism $f: X \to X$ of a compact metric space X with metric d is *expansive* (e.g., see [1] and [2]) if there is a positive number $c>0$ such that if $x, y \in X$ and $x \neq y$, then there is an integer $n=n(x, y) \in \boldsymbol{Z}$ such that

$$d(f^n(x), f^n(y)) > c .$$

A homeomorphism $f: X \to X$ is *continuum-wise expansive* [6] if there is a positive number $c>0$ such that if A is a nondegenerate subcontinuum of X, then there is an integer $n=n(A) \in \boldsymbol{Z}$ such that

$$\operatorname{diam} f^n(A) > c ,$$