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# Asymptotic expansion of an oscillating integral on a hypersurface

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## 1. Introduction.

Let $f:(\boldsymbol{R}^n, \vec{0})\to(\boldsymbol{R}, \vec{0})$ be a germ of an analytic function at $\vec{0}$ in $\boldsymbol{R}^n$ and $g:(\boldsymbol{R}^n, \vec{0})\to(\boldsymbol{R}, \vec{0})$ be a germ of an analytic function at the origin. We consider an oscillating integral of a phase function $f(\boldsymbol{x})$ with the constraint equation $g(\boldsymbol{x})=0$ such that

$$I(\tau, \varphi)=\int_{\boldsymbol{R}^n} e^{i\tau f(\boldsymbol{x})}\delta(g(\boldsymbol{x}))\varphi(\boldsymbol{x})d\boldsymbol{x} \tag{1}$$

where $\tau$ is a real parameter and $\varphi\in C_0^\infty(\boldsymbol{R}^n)$. $\delta(g(\boldsymbol{x}))$ is the "delta-function" [**3, 17**] expressing the constraint $g(\boldsymbol{x})=0$. This type of the oscillating integral sometimes appears in a kind of the path integral in physics as the Faddeev-Popov method for gauge theory ([**15**]), and also appeared in a mathematical physics (for example, see [**16**]). It is meaningful to obtain the asymptotic expansion of the integral for a large $\tau$.

The existence of the asymptotic expansion for the oscillating integral (1) is easily proved by a technique of Jeanquatier [**11**] (see also Arnold, Guzein-Zade and Varchenko [**7**]) or Malgrange [**13**]. The purpose of this paper is to calculate the principal term of the asymptotic expansion of the oscillating integral (1) in a neighborhood of a singularity of the phase in terms of Newton's diagram of the phase function and the constraint equation. The asymptotic expansion of an oscillating integral without the constraint $\delta(g(\boldsymbol{x}))$ is already studied by Varchenko [**6**] by means of the toroidal embedding. Our considerations are not included in their works. It is shown by seeing the next example. Let $f_d$ be the principal term such that $f_d=x^2y^2z^2$ in $\boldsymbol{R}^3$. We consider the hyperplane $g=x+y+z=0$. Substituting the equation into $f_d$, this term has the form $f_d=x^2y^2(x+y)^2$. This $f_d(x, y)$ is degenerate type in the meaning of Varchenko. Generally, the non-degeneracy depends on the choice of the coordinate. However in the above case we can prove easily that any coordinate transformation does not change its degeneracy. Hence if one should estimate the integral (1) with substituting directly the constraint equation $g(\boldsymbol{x})=0$, then the substituted