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Uniqueness of stable minimal surfaces with partially free boundaries

Dedicated to Robert Finn on the occasion of his seventieth birthday

By Stefan HILDEBRANDT and Friedrich SAUVIGNY

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1. Introduction.

The aim of this paper is to prove a uniqueness theorem for stable minimal surfaces $X: B \to \boldsymbol{R}^3$ of the type of the disk which are stationary in a boundary configuration $\langle \Gamma, S \rangle$ consisting of a surface S and of a Jordan arc Γ with endpoints on S. The existence of such surfaces for a prescribed configuration $\langle \Gamma, S \rangle$ was established by Courant under fairly general assumptions on Γ and S, while H. Lewy proved the first basic results on boundary regularity of minimizers. A detailed investigation of this problem with regard to existence, boundary regularity and properties of the free trace can be found in the recent monograph [**3**]; cf. also [**2**] and [**9**].

It is well-known that in general a configuration $\langle \Gamma, S \rangle$ bounds more than one stationary minimal surface of disk-type and even more than one minimizer. In fact, uniqueness seems to be a rather rare phenomenon, and not much is known about as to when it will occur. To our knowledge the question of uniqueness of minimal surfaces solving a free boundary value problem was only studied in the papers [**4**]-[**6**]. Here we want to prove a restricted uniqueness result applying only to stable minimal surfaces, whereas [**5**] and [**6**] require no restrictions of this kind. On the other hand, the method of this paper, derived from ideas of [**11**], applies to more general configurations $\langle \Gamma, S \rangle$ than [**5**] and [**6**], and also applications to H-surfaces seem possible. In [**7**] the results and techniques of this paper will be used to study existence and uniqueness for a singular problem, of which [**4**] is in some sense a limit case.

Let us now fix some notation to be used in the sequel. We denote by

$$X(u, v) = (X^1(u, v), X^2(u, v), X^3(u, v))$$

a minimal surface defined on the parameter domain $B = \{(u, v) \in \boldsymbol{R}^2 : u^2 + v^2 < 1, v > 0\}$. This is to say, $X: B \to \boldsymbol{R}^3$ is a harmonic mapping,

$$\Delta X = 0\,, \tag{1.1}$$