J. Math. Soc. Japan
Vol. 47, No. 3, 1995

On energy decay-nondecay problems for wave equations with nonlinear dissipative term in $\boldsymbol{R}^N$

By Kiyoshi MOCHIZUKI and Takahiro MOTAI

(Received Sept. 13, 1993)

1. Introduction.

In this paper we are concerned with the energy decay and nondecay problems of solutions to the wave equation

$$w_{tt}-\Delta w+\lambda w+\beta(x, t, w_t)w_t = 0, \qquad (x, t) \in \boldsymbol{R}^N \times (0, \infty) \tag{1.1}$$

with initial data

$$w(x, 0) = w_1(x) \quad \text{and} \quad w_t(x, 0) = w_2(x), \quad x \in \boldsymbol{R}^N . \tag{1.2}$$

Here $w_t=\partial w/\partial t$, $w_{tt}=\partial^2 w/\partial t^2$, Δ is the N-dimensional Laplacian, $\lambda \geqq 0$ and $\beta(x, t, w_t)w_t$ represents a dissipative term. In the following it is restricted to the power nonlinearity

$$\beta(x, t, w_t(x, t)) = b(x, t)|w_t(x, t)|^{\rho-1} \tag{1.3}$$

with $b(x, t) \geqq 0$ and $\rho>1$, or to the cubic convolution

$$\beta(x, t, w_t(x, t)) = (V_\gamma * w_t(t)^2)(x) = \int_{\boldsymbol{R}^N} V_\gamma(x-y) w_t(y, t)^2 dy \tag{1.4}$$

with $V_\gamma(x)=|x|^{-\gamma}$ $(0<\gamma<N)$. In order to guarantee regularities of solutions, we require in (1.3)

$$|b_t(x, t)|+|\nabla b(x, t)| \leqq Cb(x, t) \tag{1.5}$$

for some $C>0$, where $\nabla f=(\partial_1 f, \cdots, \partial_N f)$, $\partial_j=\partial/\partial x_j$.

We use the following notation: L^p $(1 \leqq p \leqq \infty)$ is the usual space of all L^p-functions in $\boldsymbol{R}^N$; If X is a Banach space and $I \subset \boldsymbol{R}$ is an interval, then by $C(I; X)$ and $L^p(I; X)$ we mean the space of all X-valued continuous and L^p-functions on I, respectively; H^k $(k=1, 2, \cdots)$ is the Sobolev space with norm

$$\|f\|_{H^k} = \left\{ \sum_{|\alpha| \leqq k} \int_{\boldsymbol{R}^N} |\nabla^\alpha f(x)|^2 dx \right\}^{1/2} < \infty ,$$

where α are the multi-indices; E is the space of pairs $f=\{f_1, f_2\}$ of functions