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On classification of non-Gorenstein Q-Fano 3-folds of Fano index 1

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1. Introduction.

First of all we recall some definitions.

DEFINITION 1.1. A d-dimensional normal complex projective variety X is called a *Q-Fano d-fold* if it has only terminal singularities and the anti-canonical Weil divisor $-K_X$ is ample (cf. [**KMM**]). The *index of singular point p* is defined to be the smallest positive integer i_p such that i_pK_X is a Cartier divisor near p. A singular point of singularity index one is called *Gorenstein singularity. Singularity index $I(X)$* of X is defined to be the smallest positive integer such that IK_X is a Cartier divisor. Hence there is a positive integer r and a Cartier divisor H such that $-IK_X \sim rH$. Taking the largest number of such r, we call r/I the *Fano index* of X.

Q-Fano d-folds whose Fano indices are greater than $d-2$ are classified by [**Sa**] under the assumption that they are not Gorenstein, that is, their singular indices are greater than one. In this paper we shall consider Fano 3-folds of Fano index 1 and not Gorenstein. Classifying these Fano 3-folds also answers the next problem presented by G. Fano, A. Conte and J. P. Murre (cf. [**CM**]) in the case that they have only terminal singularities.

PROBLEM. *Classify the projective* 3-*folds having Enriques surfaces as hyperplane sections.*

In general case, this problem seems very hard to solve because their singularities may not be Q-Gorenstein, that is, $-mK$ is not Cartier for any positive integer m.

In this article we shall obtain next result.

THEOREM 1.1. *Let X be a Q-Fano 3-fold of Fano index* 1 *having only cyclic quotient singularities. We take a canonical cover:*

$$Y = Spec_X \bigoplus_{m=0}^{I-1} \mathcal{O}_X(m(K_X+H)) \xrightarrow{I:1} X .$$