J. Math. Soc. Japan
Vol. 47, No. 2, 1995

Application of the theory of KM_2O-Langevin equations to the non-linear prediction problem for the one-dimensional strictly stationary time series

Dedicated to Professor Kiyoshi Ito on his seventy-seven birthday

By Yasunori OKABE and Takashi OOTSUKA

(Received Feb. 5, 1993)
(Revised Aug. 16, 1993)

§ 1. Introduction.

We are inspired by Masani-Wiener's work ([4]) of the non-linear prediction problem of a one-dimensional discrete time strictly stationary process. The purpose of the present paper is to give computable algorithms for the non-linear predictor by applying the theory of KM_2O-Langevin equations.

We have already applied in [7] the theory of KM_2O-Langevin equations to the linear prediction problem for the multi-dimensional weakly stationary time series and given a refinement of Wiener-Masani's work in [13], [14] and [3] by obtaining computable algorithms for the linear predictor. The results in [7] play supplementary but useful roles in the present approach to the non-linear problem, as will be explained.

Let $\boldsymbol{X}=(X(n); n\in\mathbb{Z})$ be a real-valued strictly stationary time series on a probability space $(\Omega, \mathcal{B}, P)$ with mean zero. We shall impose the following two hypotheses which are the same as in [4]:

(H.1) *$\boldsymbol{X}$ is essentially bounded, i.e., there exists a positive constant $C>0$ such that $|X(n)(\omega)|\leqq C$ for any $n\in\mathbb{Z}$ and almost all $\omega\in\Omega$;*

(H.2) *For any distinct integers $n_1, n_2, \cdots, n_k$ $(k\in\mathbb{N})$ the spectrum of the distribution function of the k-dimensional random variable ${}^t(X(n_1), X(n_2), \cdots, X(n_k))$ has positive Lebesgue measure.*

The non-linear predictor $\hat{X}(\nu)$ of the future $X(\nu)$, $\nu>0$, on the basis of the present and past $X(l)$, $l\leqq 0$, is defined by

$$\hat{X}(\nu) = E(X(\nu)\,|\,\sigma(X(l); l\leqq 0)).$$

Masani and Wiener ([4]) have obtained a representation for the non-linear

This research was partially supported by Grant-in-Aid for Science Research No. 03452011, the Ministry of Education, Science and Culture, Japan.