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Algorithmic methods for Fuchsian systems of linear partial differential equations

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Introduction.

A generalization of the notion of regular singularity for linear ordinary differential equations to (single) partial differential equations was introduced by Baouendi and Goulaouic [1]. They called such equations Fuchsian partial differential equations with respect to a hypersurface. In [12], [21], Kashiwara and Oshima called the same equations ones with regular singularities in a weak sense along a hypersurface and studied the boundary value problem for such equations.

Recently, the notion of Fuchsian partial differential equation of [1] has been generalized to that of Fuchsian system of linear partial differential equations along a submanifold Y of arbitrary codimension by Laurent and Monteiro Fernandes [13]. Especially, it has been proved in [13] for Fuchsian systems that any power series solution which converges with respect to the variables tangent to Y and formal with respect to the variable(s) normal to Y converges with respect to all the variables. It is also known that the holonomic system with regular singularities in the sense of Kashiwara and Kawai is Fuchsian along any submanifold (cf. [11], [13]). Thus Fuchsian systems constitute a nice and substantially wide class of systems containing many interesting examples.

Suppose that a system of linear partial differential equations

$$\mathcal{M}: P_1 u = \cdots = P_s u = 0$$

for an unknown function u in an open subset of C^{n+1} and a non-singular complex analytic hypersurface Y are given. (For example, if $\mathcal{M}$ is holonomic, then we take as Y an irreducible component of the "singular locus" of $\mathcal{M}$.) Then, from the computational point of view, we have the following basic problems about $\mathcal{M}$:

A. Is $\mathcal{M}$ Fuchsian along Y?

B. If so, find the structure of the space of multi-valued analytic (or hyper-