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A generalization of H-surfaces and a certain duality

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§1. Introduction.

In 1970 Lawson [2] showed that for a simply connected Riemann surface M there exists a bijective correspondence between minimal immersions of M into S^3 and isometric immersions of M into $\boldsymbol{R}^3$ with constant mean curvature ($\neq 0$).

As a generalization of surfaces of constant mean curvature $H\neq 0$ (in abbreviation H-surfaces), we can consider solutions of

$$\Delta f = 2H\frac{\partial f}{\partial x}\wedge\frac{\partial f}{\partial y}, \tag{1.1}$$

where (x, y) is an isothermal coordinate system,

$$\Delta f = \frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}$$

and

$$\frac{\partial f}{\partial x}\wedge\frac{\partial f}{\partial y} = \begin{pmatrix} \dfrac{\partial f^2}{\partial x}\dfrac{\partial f^3}{\partial y}-\dfrac{\partial f^2}{\partial y}\dfrac{\partial f^3}{\partial x} \\ \dfrac{\partial f^3}{\partial x}\dfrac{\partial f^1}{\partial y}-\dfrac{\partial f^3}{\partial y}\dfrac{\partial f^1}{\partial x} \\ \dfrac{\partial f^1}{\partial x}\dfrac{\partial f^2}{\partial y}-\dfrac{\partial f^1}{\partial y}\dfrac{\partial f^2}{\partial x} \end{pmatrix}$$

(see §**2**).

In this paper we shall show the following generalization of Lawson's result.

THEOREM. *Let M be a simply connected Riemann surface. Then there exists a bijective correspondence between*

$$\{\varphi : M \longrightarrow S^3 \mid \varphi \text{ is a harmonic map}\}/SO(4)$$

and

$$\{f : M \longrightarrow \boldsymbol{R}^3 \mid f \text{ satisfies (1.1)}\}/SO(3)\ltimes \boldsymbol{R}^3 .$$