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On the wellposed Cauchy problem for some dispersive equations

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1. Introduction.

The forward Cauchy problem for the operator with real coefficients $Hu(t, x) =\partial_t u(t, x)+a(x)\partial_x^2 u(t, x)+b(x)\partial_x u(t, x)+c(x)u(t, x)$ with the datum on a line $t=0$ is L^2 and H^∞-wellposed if and only if $a(x)\leqq 0$.

We consider the same problem for the operator with real coefficients

$$Au(t, x)=\partial_t u(t, x)+\partial_x^3 u(t, x)+a(x)\partial_x^2 u(t, x)+b(x)\partial_x u(t, x)+c(x)u(t, x). \tag{1.1}$$

which is obtained by adding the dispersive term $\partial_x^3 u(t, x)$ to $Hu(t, x)$. Our problem is under which conditions on the coefficient $a(x)$ the forward Cauchy problem for $Au(t, x)$ is L^2 or H^∞-wellposed.

Similar problems arise for the Schrödinger type operator

$$Su(t, x)=\partial_t u(t, x)+i\partial_x^2 u(t, x)+A(x)\partial_x u(t, x)+B(x)u(t, x).$$

In this case, the following condition on the imaginary part of $A(x)$: $\Im A(x)$ is necessary and sufficient for the L^2[resp. H^∞]-wellposedness;

There exists some constant C satisfying

$$\left|\int_x^y \Im A(x)dx\right| \leqq C \quad \left[\text{resp.} \left|\int_x^y \Im A(x)dx\right| \leqq C\log(|x-y|+2)\right]$$
$$\text{for any} \quad x,\ y\in \boldsymbol{R},$$

while for the operator $\partial_t u(t, x)+A(x)\partial_x u(t, x)+B(x)u(t, x)$ the necessary and sufficient condition is $\Im A(x)=0$ (see W. Ichinose [1] and [2], S. Mizohata [4] and J. Takeuchi [6]).

In the following, we consider only real-valued functions and operators with real coefficients with some obvious exceptions.

Now we formulate the forward Cauchy problem for the operator A defined by (1.1):

For the given datum $g(x)$ and right-hand side $f(t, x)$ of the equation, find a solution $u(t, x)$ satisfying