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Normality of affine toric varieties associated with Hermitian symmetric spaces

By Mutsumi SAITO

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§0. Introduction.

Let an algebraic torus T of dimension n act on a vector space V of dimension N $(N>n)$ via N characters $\chi_1, \cdots, \chi_N$ of T. We assume the above characters to generate the character group $X(T)$ of T and to lie on one hyperplane of $\boldsymbol{R}\otimes_{\boldsymbol{Z}} X(T)$. Let A be the polynomial ring $\boldsymbol{Z}[\xi_1, \cdots, \xi_N]$, and let L be the subgroup of $\boldsymbol{Z}^N$ consisting of the elements $a=(a_j)_{1\leq j\leq N}$ such that $\sum_{j=1}^N a_j\chi_j=0$. We consider the ring

$$R = A/\sum_{a\in L} A\xi_a .$$

Here $\sum_{a\in L} A\xi_a$ denotes the ideal of A consisting of all sums $\sum_{a\in L} p_a\xi_a$ with $p_a\in A$ where $\xi_a=\prod_{a_j>0}\xi_j^{a_j}-\prod_{a_j<0}\xi_j^{-a_j}$, and only finitely many p_a are not zero. In this situation Gelfand and his collaborators studied generalized hypergeometric systems (cf. [**G**], [**GGZ**], [**GZK1**], [**GZK2**], [**GKZ**]). We notice that the idea of this kind of generalized hypergeometric systems goes back to [**H**] and [**KMM**]. We remark that Aomoto also defined and studied generalized hypergeometric functions by use of integral representations (cf. [**A1**]-[**A4**]). We can find in [**GZK2**] the computation of the characteristic cycles of generalized hypergeometric systems; we cannot follow this computation unless the Z-algebra R is normal, however. In [**S**] we defined the b-functions of generalized hypergeometric systems, and used the normality of the $\boldsymbol{Z}$-algebra R in order to determine those b-functions. Hence the normality of the $\boldsymbol{Z}$-algebra R is very important.

In this paper we assume V to be an open Schubert cell of a simple compact Hermitian symmetric space and T to be a maximal torus of its motion group. We remark that the generalized hypergeometric system corresponding to the Lauricella function F_C, and the one to the Lauricella function F_D are defined in this setup (cf. [**GZK2**]). Then we prove

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