J. Math. Soc. Japan
Vol. 46, No. 4, 1994

On Keen's moduli inequality in two generator Möbius groups

By Takehiko SASAKI

(Received Oct. 16, 1992)
(Revised April 21, 1993)

§1. Introduction and statement of results.

In general, it is difficult to determine whether a two generator Möbius group is Kleinian or not, or is discrete or not. In [**1**], for the purpose of studying one dimensional Teichmüller spaces, Keen obtained a moduli inequality which assures some two generator Möbius groups are Kleinian. To state her theorem, we need some notation. Let A and B be Möbius transformations and let $G=\langle A, B\rangle$ be the group generated by A and B. By the well known isomorphism between the Möbius group and $PSL(2, \boldsymbol{C})$, we put

$$x=\operatorname{trace}(A), \quad y=\operatorname{trace}(B) \quad \text{and} \quad z=\operatorname{trace}(AB).$$

The groups we are interested in this article are those which satisfy the following.

$$x^2+y^2+z^2=xyz, \tag{1}$$

$$z>2 \quad \text{and} \tag{2}$$

$$|x|>2 \quad \text{and} \quad |y|>2. \tag{3}$$

For those groups Keen showed the following.

THEOREM 1 ([**1**]). *If the moduli triple* (x, y, z) *satisfies* (1), (2), (3) *and the inequality*

$$|z\operatorname{Im}(x)-2\operatorname{Im}(y)|<2|\operatorname{Re}(x)|, \tag{4}$$

then the group $G=\langle A, B\rangle$ *is Kleinian.*

On the other hand, we showed the following.

THEOREM 2 ([**3**]). *Let* $U=\begin{pmatrix}\alpha & 0\\ 0 & \beta\end{pmatrix}$ *and* $V=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$, $bc\neq 0$, *be loxodromic elements of* $PSL(2, \boldsymbol{C})$ *such that* $UVU^{-1}V^{-1}$ *is parabolic. If, for each integer* n, *the inequality*

$$\frac{|\alpha^n a|+|\beta^n d|}{|\alpha^n a+\beta^n d|}<\frac{|\alpha|+|\beta|}{|\alpha-\beta|} \tag{5}$$