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Asymptotic profiles of blow-up solutions of the nonlinear Schrödinger equation with critical power nonlinearity

Dedicated to Professor R. Iino on his 70th birthday

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1. Introduction.

We shall consider the blow-up problem for the nonlinear Schrödinger equation:

$$\text{C(p)} \qquad \begin{cases} \text{(NS)} & 2i\dfrac{\partial u}{\partial t}+\Delta u+|u|^{p-1}u=0 \qquad (t,\ x)\in \boldsymbol{R}_+\times \boldsymbol{R}^N, \\ \text{(IV)} & u(0,\ x)=u_0(x), \qquad x\in \boldsymbol{R}^N. \end{cases}$$

Here $i=\sqrt{-1}$, $u_0\in H^1(\boldsymbol{R}^N)$ and Δ is the Laplace operator on $\boldsymbol{R}^N$. The nonlinear Schrödinger equation of the form (NS) arises in various domains of physics, *e.g.*, fluids, plasmas and optics. The equation (NS) also derived from a field equation for a quantum mechanical nonrelativistic many body system in the semi-classical limit.

The unique local existence of solutions of C(p) is well known for $1<p<2^*-1$ ($2^*=2N/(N-2)$ if $N\geqq 3$, $=\infty$ if $N=1, 2$): For any $u_0\in H^1(\boldsymbol{R}^N)$, there exists a unique solution $u(t, x)$ of C(p) in $C([0, T_m); H^1(\boldsymbol{R}^N))$ for some $T_m\in(0, \infty]$ (maximal existence time), and $u(t)$ satisfies the following two conservation laws of L^2 and the energy:

$$\|u(t)\| = \|u_0\|, \tag{1.1}$$

$$E_{p+1}(u(t)) \equiv \|\nabla u(t)\|^2-\frac{2}{p+1}\|u(t)\|_{p+1}^{p+1}=E_{p+1}(u_0), \tag{1.2}$$

for $t\in[0, T_m)$, where $\|\cdot\|$ and $\|\cdot\|_{p+1}$ denotes the L^2 norm and L^{p+1} norm respectively. Furthermore $T_m=\infty$ or $T_m<\infty$ and $\lim_{t\to T_m}\|\nabla u(t)\|=\infty$. For details, see *e.g.*, [**11, 12, 14**].

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