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Fatou sets in complex dynamics on projective spaces

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Introduction.

The theory of complex dynamical systems defined by holomorphic maps on complex projective spaces, which generalizes the iteration theory of one variable rational functions, has been studied by several authors [**FS1**], [**FS2**], [**HP**], [**U3**]. Concerning Julia sets and Fatou sets, analogies to the one variable case are pursued to some extent. There are also many problems which we encounter first in higher dimensional case.

In this paper, we prove two fundamental results on Fatou sets for complex dynamical systems of degree greater than 1 on complex projective spaces: Fatou sets are pseudoconvex, hence Stein (Theorem 2.3); Fatou sets are Carathéodory hyperbolic, hence Kobayashi hyperbolic (Theorems 2.5 and 2.6). With the latter theorem, we can derive some results analogous to the one dimensional case. It is proved that the immediate basin of an attractive periodic point contains critical points. The same result is proved for a parabolic periodic point in two dimensional case under an additional condition.

To prove the above fundamental theorems we employ the method originated by Hubbard and Papadopol [**HP**]. Namely we consider, for a holomorphic map f on $\boldsymbol{P}^n$, the corresponding homogeneous polynomial map F on $\boldsymbol{C}^{n+1}$ and the Green function h with respect to F. It is shown in Theorem 2.2 that a point $p \in \boldsymbol{P}^n$ is in the Fatou set if and only if the Green function is pluriharmonic in a neighborhood of the fiber $\pi^{-1}(p)$ of the projection $\pi : \boldsymbol{C}^{n+1} - \{O\} \to \boldsymbol{P}^n$. The "only if" part is proved in [**HP**, Proposition 5.4] and the "if" part provides the answer to a problem posed in [**HP**, below Prop. 5.4].

The outline of the present paper is as follows: In section 1, properties of homogeneous maps and Green functions are described. Using these, we prove the two main theorems on Fatou set in section 2. As applications of the hyperbolicity, results on the critical points in the Fatou sets are proved in section 3. Although the results on Green functions are due to [**HP**], we include the proofs for the sake of completeness.

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