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V-sufficiency from the weighted point of view

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Two germs of functions $f, g: (\boldsymbol{R}^n, 0) \to (\boldsymbol{R}^p, 0)$ are said to have the same (local) v-type at 0 (v stands for variety), if the germs at 0 of $f^{-1}(0)$ and $g^{-1}(0)$ are homeomorphic. Let $f: (\boldsymbol{R}^n, 0) \to (\boldsymbol{R}^p, 0)$ be a C^k-function. A very interesting problem is to determine what terms from the Taylor expansion at 0, may be omitted without changing the v-type determined by f. For a solution of this problem see [$\mathbf{K}_1$].

In this paper we shall consider the weighted analogue to this problem, and using a new singular Riemannian metric on $\boldsymbol{R}^n$(introduced in [**P**]) we shall give a characterization of v-sufficiency (Theorem A and Theorem B below). Moreover we shall give a geometric corollary for functions whose components are the sum of at most two weighted homogeneous polynomials (generalizing the case with nondegenerate weighted homogeneous components), and also we give a generalization of a well-known inequality due to Bochnak and Lojasiewicz. The use of singular Riemannian metrics seems to be quite useful, see for instance [**Y**], [**P**].

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§ 1. The results.

Let us denote by $\boldsymbol{E}(n, p)$ the set of all germs of functions $f: (\boldsymbol{R}^n, 0) \to (\boldsymbol{R}^p, 0)$ which are C^2 in a punctured neighbourhood of the origin. From now on we shall fix a system of positive numbers $w=(w_1, \cdots, w_n)$, the weights of variables x_i, $w(x_i)=w_i$, $1 \leqq i \leqq n$, and a positive number d. For any positive number q we may introduce (see [**P**]) the function $\rho=\rho(x)=(\sum_{i=1}^n x_i^{2q_i})^{1/2q}$, where $q_i=q/w_i$, $1 \leqq i \leqq n$. This is a w-form of degree one with respect to w, and if $q_i \geqq 1$, $1 \leqq i \leqq n$, then $\rho \in \boldsymbol{E}(n, 1)$. We also consider the spheres associated to this ρ

$$S_r = \{x \in \boldsymbol{R}^n \mid \rho(x) = r\}, \qquad r > 0.$$

DEFINITION 1. We define a singular Riemannian metric on $\boldsymbol{R}^n$ by the fol-