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Holomorphic maps of projective algebraic manifolds into compact C-hyperbolic manifolds

Dedicated to Professor Nobuyuki Suita on his 60th birthday

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Introduction.

Let $\mathrm{Hol}(M, N)$ be the Douady space of compact complex manifolds M and N, that is $\mathrm{Hol}(M, N)$ is the set of all holomorphic maps of M into N. Then $\mathrm{Hol}(M, N)$ has a complex analytic space structure whose underlying topology is the compact-open topology. Moreover, the evaluation map of $\mathrm{Hol}(M, N)\times N$ into N sending (f, p) to $f(p)$ is holomorphic. (See Douady [**2**].)

The main purpose of this paper is to study concretely the structure of $\mathrm{Hol}(M, N)$ for a projective algebraic manifold M and a compact C-hyperbolic manifold N. A complex manifold N is said to be C-hyperbolic or Carathéodory hyperbolic if there exists a regular covering $\tilde{N}$ of N whose Carathéodory pseudo-distance is actually a distance (see Kobayashi [**12**], p. 129). A typical example of C-hyperbolic manifolds is a quotient space $N=\Omega/\Gamma$, where Ω is a bounded domain in the n-dimensional complex Euclidean space $\boldsymbol{C}^n$ and Γ is a fixed-point-free discrete subgroup of the analytic automorphism group $\mathrm{Aut}(\Omega)$ of Ω. Every submanifold of a C-hyperbolic manifold is also C-hyperbolic.

Throughout this paper, we assume that M is a projective algebraic manifold over the complex number field $\boldsymbol{C}$, and N is a compact C-hyperbolic manifold. (By Noguchi and Sunada [**19**], Lemma 2.3, for a C-hyperbolic projective algebraic manifold N, it is sufficient to only assume that M is a compact complex space.) Since a compact C-hyperbolic manifold N is complete hyperbolic, $\mathrm{Hol}(M, N)$ is a compact complex analytic space with finitely many irreducible components (see Kobayashi [**12**], Theorem 3.2 in Chap. V).

In Section 1, we obtain the following main result:

THEOREM 1. *Let M be a projective algebraic manifold with universal covering transformation group G, and let N be a compact C-hyperbolic manifold with universal covering transformation group Γ. If holomorphic maps $f_1, f_2: M\to N$ induces the same surjective monodromy $(\tilde{f}_1)_*=(\tilde{f}_2)_*: G\to\Gamma$ and if $f_1(M)\cap f_2(M)$*