J. Math. Soc. Japan
Vol. 46, No. 2, 1994

Asymptotic behavior of positive solutions of singular Emden-Fowler type equations

Dedicated to Professor Takaŝi Kusano on his 60th birthday

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(Received July 13, 1992)
(Revised Oct. 23, 1992)

1. Introduction.

This paper is concerned with positive solutions of the singular Emden-Fowler type equation

$$x'' = p(t)x^{-\lambda}, \qquad t \geqq t_0 > 0, \tag{1.1}$$

where $\lambda>0$, and p is a positive continuously differentiable function on $[t_0, \infty)$. By a positive solution of (1.1) we mean a positive function $x(t)$ of class C^2 which solves (1.1) for $t \geqq t_0$.

Let $x(t)$ be a positive solution of (1.1). Then, it is easily seen that $x(t)$ has exactly one of the next properties:

$$x'(t) < 0 \qquad \text{for all } t, \text{ and } \lim_{t\to\infty} x(t) \in [0, \infty), \tag{1.2}$$

or

$$x'(t) > 0 \qquad \text{for all large } t, \text{ and } \lim_{t\to\infty} \frac{x(t)}{t} \in (0, \infty]. \tag{1.3}$$

A positive solution $x(t)$ of (1.1) is called a positive decaying solution if (1.2) is satisfied with $\lim_{t\to\infty} x(t)=0$, and is called a positive increasing solution if (1.3) is satisfied.

Singular equations of this kind appear in many branches of mathematical physics. We refer the reader to the papers [**3**, **4**, **5**] for physical aspect of equation (1.1). Interesting results have been obtained for these equations; see [**1**, **7**, **8**]. Sufficient, or necessary conditions for the existence of positive solutions satisfying (1.2), in particular, positive decaying solutions, were discussed in [**8**, **9**]. However, it seems that very little is known about the asymptotic behavior as well as the uniqueness of positive decaying solutions. Our first objective is to investigate these problems. We discuss in Section 2 the asymptotic behavior of positive decaying solutions. Uniqueness criteria for positive decay-