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On the existence of Yang-Mills connections by conformal changes in higher dimensions

By Minyo KATAGIRI

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1. Introduction.

Let (M, g) be a smooth closed Riemannian n-manifold, and P a principal G-bundle over (M, g) with compact Lie group G. Let $\mathcal{C}$ be the space of all smooth connections on $P \rightarrow M$ compatible with the metric on P. The space $\mathcal{C}$ is an affine space. For a connection $A \in \mathcal{C}$, we denote by d_A and δ_A the covariant exterior derivative and its formal adjoint respectively.

The Yang-Mills functional $J: \mathcal{C} \rightarrow \boldsymbol{R}$ is defined by

$$J(A)=\frac{1}{2}\int_{M}|F_{A}|^{2}d\mu_{g}\,, \tag{1}$$

where F_A is the curvature of a connection A, and $d\mu_g$ is the volume element induced from a Riemannian metric g. A is called a *Yang-Mills connection* if A is a critical point of the Yang-Mills functional J, that is, A satisfies the Yang-Mills equation

$$\delta_A F_A = 0\,, \tag{2}$$

which is the Euler-Lagrange equation for the Yang-Mills functional J.

In dimension 4, it is well-known that the Yang-Mills functional is invariant under conformal changes of metric on the base manifold, but in general it is not invariant under this action.

In this paper, using the above fact we will show "some existence" result for Yang-Mills connections in higher dimensions:

THEOREM 1.1. *Let (M, g) be a smooth closed Riemannian manifold of dimension $n \geqq 5$, P a smooth principal G-bundle over (M, g) with compact group G. Then there exists a connection $A_0 \in \mathcal{C}$ and a metric $\tilde{g}$ on M which is conformally equivalent to the original metric g such that A_0 is a Yang-Mills connection on the principal bundle P over $(M, \tilde{g})$.*

The idea of a proof of this Theorem here is essentially due to J. Eells-M. J. Ferreira for harmonic maps ([1]). In section 2, we will state some known