J. Math. Soc. Japan
Vol. 45, No. 3, 1993

Characteristic Cauchy problems for some non-Fuchsian partial differential operators

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(Received Dec. 2, 1991)
(Revised July 30, 1992)

§ 0. Introduction.

We consider the characteristic Cauchy problems for some non-Fuchsian partial differential operators with real-analytic coefficients. We give some theorems that are similar to the Cauchy-Kovalevskaya theorem and the Holmgren theorem. As a corollary, we obtain some results on the non-existence of null-solutions.

First, we give the definition of (essentially) Fuchsian operators and null-solutions. Consider a partial differential operator

$$P=t^{k}\partial_{t}^{m}+\sum_{j+|\alpha|\leqq m,\, j<m} a_{j,\alpha}(t,\ x)\partial_{t}^{j}\partial_{x}^{\alpha}\ ,$$

where k is a non-negative integer and $a_{j,\alpha}$ are smooth (that is, C^{∞} or holomorphic etc.) in a neighborhood of $(0,0)\in \boldsymbol{R}_{t}\times\boldsymbol{R}_{x}^{n}$.

DEFINITION 0.1. (1) A partial differential operator P is called *Fuchsian* (with respect to the hypersurface $\Sigma:=\{(t,\ x)\,;\,t=0\}$), if P is written in the form

$$\begin{aligned}P=\ & t^{k}\partial_{t}^{m}+a_{m-1}(x)t^{k-1}\partial_{t}^{m-1}+\cdots+a_{m-k}(x)\partial_{t}^{m-k}\\ &+\sum_{p<m}\ \sum_{|\beta|\leqq m-p} t^{\alpha(p,\beta)}\partial_{t}^{p}a_{p,\beta}(t,\ x)\partial_{x}^{\beta}\end{aligned}$$

with $\alpha(p,\ \beta)=\max(0,\ k+p-m+1)$, where $0\leqq k\leqq m$ and a_{j}, $a_{p,\beta}$ are smooth ([1]). If P can be written as $P=t^{h}Q$, where Q is Fuchsian and h is an integer, then P is called *essentially Fuchsian* ([8]). Also, an operator P is called *non-Fuchsian* (resp. *essentially non-Fuchsian*), if P is not Fuchsian (resp. not essentially Fuchsian).

(2) A distribution u in a neighborhood of $(0,0)$ is called a *null-solution* for P at $(0,0)$ with respect to Σ (or rather $\Sigma_{+}=\{t>0\}$), if $Pu=0$ in a neighborhood of $(0,0)$ and $(0,0)\in\operatorname{supp} u\subset\{t\geqq 0\}$.

After a pioneering study by Y. Hasegawa ([4]), M. S. Baouendi and C. Goulaouic ([1]) defined Fuchsian partial differential operators, and proved some generalizations of the classical Cauchy-Kovalevskaya theorem and the Holmgren