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On the space of self homotopy equivalences of the projective plane

By Tsuneyo YAMANOSHITA

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1. Introduction.

Let X be a connected CW complex with base point, and let $G(X)$ and $G_0(X)$ be the spaces of self homotopy equivalences of X and self homotopy equivalences of X preserving the base point respectively.

It seems that little is known about the relation between $G(X)$ and X except an H-space X (see [**10**]). When X is not an H-space, we have Gottlieb's theorem on $G(K(\pi, 1))$ ([**4**]) and Hansen's theorem on $G(S^2)$ ([**7**]).

Of the projective plane P^2, it is known that the spaces $\mathrm{Top}(P^2)$ and $\mathrm{Diff}(P^2)$ both have the same homotopy type as $SO(3)$, where $\mathrm{Top}(P^2)$ and $\mathrm{Diff}(P^2)$ are the space of homeomorphisms of P^2 and the space of diffeomorphisms of P^2 respectively. This was proved for $\mathrm{Top}(P^2)$ by M.-E. Hamstrom [**5**] in 1965 and for $\mathrm{Diff}(P^2)$ by C. J. Earle and J. Eells [**3**] in 1969.

In this paper we shall prove the following

THEOREM. *There is a homeomorphism*

$$G(P^2) \cong SO(3) \times (G_0(P^2)/O(2)).$$

2. Compact Lie group actions.

Throughout this paper, all spaces will be Hausdorff spaces with base points when necessary and all spaces of maps will be equipped with the compact open topology.

For Lie group actions on manifolds, we have the following

PROPOSITION 1. *Let L be a compact Lie group which transitively acts on a connected closed manifold M. And let S be the isotropy subgroup at the base point. Then there is a homeomorphism*

$$G(M) \cong L \times_S G_0(M).$$

PROOF. We begin by defining an L-action on $G(M)$ by $\sigma \cdot f = \sigma \circ f$ for $\sigma \in L$,