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# An approach to the characteristic free Dutta multiplicity

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## Introduction.

In "Algèbre locale. Multiplicités" [20], Serre conjectured:

CONJECTURE. *Let $X$ be a connected regular scheme and $Y$, $Z$ closed irreducible subschemes of $X$. Then, for each irreducible component $W$ of $Y \cap Z$,*

(S1) $\operatorname{codim}(W, X) \leqq \operatorname{codim}(Y, X) + \operatorname{codim}(Z, X)$,

(S2) *if* $\operatorname{codim}(W, X) < \operatorname{codim}(Y, X) + \operatorname{codim}(Z, X)$, *then*

$$\sum_i (-1)^i l_{\mathcal{O}_{W,X}}(\operatorname{Tor}_i^{\mathcal{O}_{W,X}}(\mathcal{O}_{W,Y}, \mathcal{O}_{W,Z})) = 0\,,$$

(S3) *if* $\operatorname{codim}(W, X) = \operatorname{codim}(Y, X) + \operatorname{codim}(Z, X)$, *then*

$$\sum_i (-1)^i l_{\mathcal{O}_{W,X}}(\operatorname{Tor}_i^{\mathcal{O}_{W,X}}(\mathcal{O}_{W,Y}, \mathcal{O}_{W,Z})) > 0\,.$$

In [20] Serre proved (S1) in general and (S2), (S3) in the case where the regular local ring $\mathcal{O}_{W,X}$ is unramified, i.e., either $\mathcal{O}_{W,X}$ contains a field or the square of its maximal ideal does not contain $p$, where $p>0$ is the characteristic of the residue class field $\mathcal{O}_{W,X}/\mathcal{M}_{W,X}$. Furthermore Roberts [17], Gillet and Soulé [9] independently solved (S2) affirmatively. (Roberts proved (S2) under a weaker condition ([15], [16], [17]) using the intersection theory. Dutta, Hochster and MacLaughlin [6] constructed the following important example:

Put $A=k[[x, y, z, w]]/(xy-zw)$ ($k$ is a field), and $M=A/(x, z)$. Then there exists an $A$-module $N$ such that $l_A(N)=15$, $pd_A N=3$ and $\sum_i(-1)^i l_A(\operatorname{Tor}_i^A(M, N))=-1$.
We can explain this phenomenon in terms of localized Chern characters as in Example 18.3.14 in [8].)

(S1) is a remarkable result which enables us to estimate the minimum of the dimension of the intersection of two closed irreducible subschemes when they actually intersect. It is expected that such an inequality holds even under a weaker condition. (By the intersection theorem due to Roberts [18], we have

$$\dim M + \operatorname{depth} N \leqq \operatorname{depth} A$$

for any Noetherian local ring $A$ and any finitely generated $A$-modules $M$ and