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On strong C^0-equivalence of real analytic functions

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Let $\mathcal{E}_{[\omega]}(n, 1)$ be the set of analytic function germs: $(\boldsymbol{R}^n, 0)\rightarrow(\boldsymbol{R}, 0)$. In [7]-[12], T.C. Kuo has introduced some notions of blow-analycity as desirable (natural) equivalence relations for elements of $\mathcal{E}_{[\omega]}(n, 1)$, and has given important results to construct blow-analytic theory. Stimulated by his work, several singularists started on studying blow-analycity and introduced notions similar to but different from those of Kuo's ([2], [3], [5], [6], [16], [18]). These blow-analytic equivalences are slightly weaker than bianalycity, and much stronger than homeomorphism. In this note, we introduce the notion of strong C^0-equivalence as one of blow-analytic equivalences. Roughly speaking, it is a C^0-equivalence which preserves the tangency of analytic arcs at $0\in\boldsymbol{R}^n$. It seems that this equivalence is not so strong. In fact, this is weaker than some other blow-analytic equivalences. Our purposes in this paper are to formulate two conditions which imply strong C^0-equivalence and to show the Briançon-Speder family ([1]) and the Oka family ([15]) are not strongly C^0-trivial.

In the complex case, the Briançon-Speder family is well-known as an example that topological triviality does not imply the Whitney regularity, in other words, the Milnor number constancy does not imply μ^*-constancy. The Oka family also is μ-constant but not μ^*-constant. Both families have a *weak simultaneous resolution*, but have no *strong simultaneous resolution* (see [13], [14], [15], [17]). In the real case, however, the families are topologically trivial, but not even strongly C^0-trivial.

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§1. Results.

At first we define the notion of strong C^0-equivalence.

NOTATION. (1) By an analytic arc at $0\in\boldsymbol{R}^n$, we mean the germ of an analytic map $\lambda: [0, \varepsilon)\rightarrow\boldsymbol{R}^n$ with $\lambda(0)=0$, $\lambda(s)\neq 0$, $s>0$. The set of all such arcs

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