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# Hodge-Witt cohomology of complete intersections

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## 1. Statement of the theorem.

In this note, we prove the following assertions.

THEOREM. *Let $k$ be a perfect field of characteristic $p>0$ and $X$ a smooth complete intersection of dimension $n$ in a projective space over $k$.*

(a) *If $i\neq j$ and $i+j\neq n$, $n+1$, $H^{j}(X, W\Omega_{X}^{i})=0$.*

(b) *If $2i\neq n$, $n+1$ and $0\leqq i\leqq n$, $H^{i}(X, W\Omega_{X}^{i})=W$ and $F$ is bijective on $H^{i}(X, W\Omega_{X}^{i})$.*

(c) *$H^{n-i}(X, W\Omega_{X}^{i})$ is a Cartier module (in the sense of* [**5**], Ch. I, Def. 2.4).

(d) *If $2i\neq n+1$, $H^{n-i+1}(X, W\Omega_{X}^{i})/F^{\infty}B=0$.*

(e) *If $2i=n+1$, $H^{n-i+1}(X, W\Omega_{X}^{i})/F^{\infty}B=W$ and $F$ is bijective on $H^{n-i+1}(X, W\Omega_{X}^{i})/F^{\infty}B$.*

We follow the notation of [**1**], [**4**] and [**5**]. In particular, $W=W(k)$ (resp. $K$) is the ring of Witt vectors with coefficients in $k$ (resp. the fraction field of $W$). $H^{\cdot}(X/W)$ (resp. $H^{j}(X, W\Omega_{X}^{i})$) denotes the crystalline cohomology group (resp. the Hodge-Witt cohomology group) of $X$. $F$ (resp. $V$) stands for the Frobenius morphism (resp. the Verschiebung morphism). For a commutative group $A$ and an endomorphism $m$ of $A$, ${}_{m}A$ (resp. $A/m$) denotes $\mathrm{Ker}[m: A\to A]$ (resp. $\mathrm{Coker}[m: A\to A]$).

## 2. Proof of the theorem.

Throughout this section, $k$ denotes a perfect field of characteristic $p>0$ and $X$ a smooth complete intersection of dimension $n$ in a projective space over $k$.

We first recall known facts on the Hodge cohomology and the crystalline cohomology of a smooth complete intersection in a projective space:

(I) $H^{j}(X, \Omega_{X}^{i})=0$ if $i\neq j$ and $i+j\neq n$;

(II) $H^{i}(X, \Omega_{X}^{i})=k$ if $2i\neq n$ and $0\leqq i\leqq n$;

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